Service Product Design and Consumer Refund Policies

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We consider the product line design problem of a monopolistic firm selling to a heterogeneous customer population, taking into consideration the possibility that customers may return their purchases for a refund. We show that a wide range of product line designs and refund policies can emerge, which is in line with the myriad of refund terms observed in practice, even within the same industry. The key driver lies in the characteristics of customers' valuation uncertainty. In particular, customer refunds can induce *variety reduction* in the product line — even with customization capabilities, the firm may still choose to offer a single quality level or a standard refund rate across products. This is in stark contrast to situations without customer refunds, where a full product line is always desirable. In addition, partial refunds can be optimal in our setting, which does not involve aggregate demand uncertainty, capacity limitations, competition, or channel conflicts. Interestingly, despite its appeal, generous refund terms do not increase aggregate customer surplus. Furthermore, the firm may not have incentives to reduce customers' valuation uncertainty even if doing so is costless. Our findings bear practical implications across a variety of service domains.

Key words: product line design; cancellations; return policy; partial refunds; consumer uncertainty; services; quality

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1. Introduction

Firms often tailor their products to customers in many service industries.[1] For instance, Comcast adjusts the speed of Internet services or TV channel lineups based on the packages purchased by customers. As another example, airlines divide seats into business and coach compartments, each with different levels of service. In the meantime, customers often face uncertainties with respect to their service needs and product valuations; as a result, firms need to make decisions regarding their cancellation and refund terms. In practice, a wide variety of product line design and refund policies are observed, even within the same industry. A prime example is the airline industry. Business-class customers are offered premium services and fully refundable rates, while economy-class seats are

[1] We use "product" and "service" interchangeably throughout the paper.

often sold at many fare classes with different refund terms. In addition, opaque products can be offered at relatively low but non-refundable rates. Even though refunds are commonly available in many industries, they can be quite restrictive for others. Consider concerts, special events, and theme parks for instance. Disney World tickets and packages, despite their numerous options for admissions and in-park services, are all non-transferable and non-refundable. At one of the authors' local symphony orchestra, ticket prices for an event range from \$34 to over \$200 depending on the seating zones, thus exhibiting substantial quality difference. Yet, all of these tickets are non-refundable.

Consumer refunds and service cancellations have each been well studied in the marketing and operations literature. Much of this literature, however, considers refund policies under a *given* product line, and in many cases assumes that a single product or quality level is offered. A separate stream of marketing and operations literature has centered on the issue of product customization, or product line design. However, little research in this area has considered the existence of refund policies.

In this paper, we aim to bridge these gaps by studying the interaction between product design and refund policies. An intriguing research question is what the particular mechanisms are that drive the myriad of product lines and refund policies in practice. When customers make product choices, they apparently will take into account the refund terms associated with each product. It is also reasonable to expect that customers across different market segments (such as business vs. leisure travellers) and industries are distinct. Therefore, we are interested in investigating how market characteristics may affect a firm's product line and refund policy design.

In exploring these questions, we consider the following setup: a monopolistic firm sells to a customer population with heterogeneous valuation on product quality. Customers do not observe their valuations before the purchase but receive an imperfect signal indicating their types. The firm decides on its product qualities, prices, and refund policies. Consumers self-select the product to purchase based on the signal they receive. After the purchase, customers discover their true types and decide whether to return the product and obtain a refund.

It is well understood in the literature that returns may result from customers' product valuation uncertainty: customers experience uncertainty prior to product purchase and make returns if their valuations turn out to be low after their purchase. Customer returns in our model are also driven by valuation uncertainty. Different from much of the existing literature, however, we explicitly model customers' valuation uncertainty in two aspects. The first aspect involves valuation heterogeneity: some customers have high valuations, while the rest have low valuations. This aspect captures the uncertainty on the *aggregate* level, and can be measured by the magnitude of the relative valuation difference. The second aspect is customers' knowledge regarding their own valuations: customers may receive a strong or weak signal about their valuations. Thus on the *individual* level, a customer may be less or more certain about her valuation depending on the signal strength. We show that both aspects are important drivers of our results.

We fully characterize the firm's optimal product line with an endogenous refund policy. As a baseline, we consider the case where refunds are not allowed and show that it is optimal to offer quality-differentiated products to different customer segments. In other words, a full production line is always desirable in the absence of refund. However, when refunds are allowed, it can be optimal for the seller to offer products with the same quality to different customer segments, albeit with different refund terms. Therefore, the flexibility to customize refund policies for different customer segments leads to a *variety reduction* effect. In general, the optimal product line depends on the nature of customer uncertainty, which is characterized by the valuation difference across customer segments and by how informed customers are about their true types. Specifically:

1. When the valuation difference across customer segments is large, it is optimal for the seller to offer fully-refundable premium-quality products to all customers.
2. When customers show moderate valuation differences and are not well-informed, the seller should offer non-refundable good-quality service to customers who are optimistic about their valuations, and fully-refundable premium-quality service to pessimistic customers.
3. When customers show moderate valuation differences but are well-informed, the seller should instead offer a premium product with a partial refund to the optimistic customers and a non-refundable low-quality product to the pessimistic customers.
4. When the valuation difference across customer segments is small, the seller should adopt a no-refund policy while providing high- and low-quality products to the optimistic and pessimistic customers, respectively.

The results above seem to explain many phenomena observed in the marketplace. To understand Result 1, we note that a large valuation difference can be driven by customer needs. For example, a customer who does not have a true need for a product will place a low value on the product. Business travellers sometimes need to cancel their airline tickets and hotel reservations due to schedule conflicts or business emergencies. Therefore, they exhibit drastic valuation differences for a product, depending on the type. This observation explains why most the offerings extended to business-class travellers and conference hotel guests are of premium quality and are fully refundable.

Result 2 corresponds quite well to leisure travel. Leisure travellers' choices among products are frequently driven by their preferences for certain product attributes. Therefore, their valuation difference is usually more moderate than that of business travellers. In addition, leisure customers are usually less-experienced and therefore often less-informed about their travel needs. As a result, they are often offered a fully refundable option, as well as a lower, but non-refundable rate. A concrete example is the economy class offerings at Air Canada. The Economy Tango is non-refundable and costs extra for same-day ticket changes, while the Economy Latitude is fully refundable with free same-day ticket changes, and comes with numerous additional services such as priority check-in and complementary checked baggage, which are not available for Economy Tango.

The difference in Result 3, compared to Result 2, is that customers are more well-informed, which often appears among home services, such as wireless service. Heavy users usually subscribe to unlimited services which can be cancelled with reasonable termination fees. Infrequent users, on the other hand, are more likely to opt for prepaid or pay-to-go service that is usually non-refundable.

Notably, in Result 3, partial refunds as a standard policy can emerge as an optimal policy,[2] which does not involve aggregate demand uncertainty, capacity limitations, competition, or channel conflicts — which are common causes identified in the existing literature for partial refunds. In our setting, partial refunds are solely driven by a moderate valuation difference across customers who are well-informed about their own type. Since these customers are better informed, it is more efficient to use quality differentiation to discriminate the customers. Compared to full refunds, proper partial refunds discourages some customers from exercising the refund and thus increases the firm's revenue.

Result 4 states that no refunds shall be offered when customers' valuation difference is small. This rule applies to most commodity products. However, even for products that are usually considered as non-commodities, customers may not be offered refunds if the valuation difference among customers is not large. This can be the case for hard core music or sports fans, which may explain why sports and concert events usually offer only non-refundable tickets, despite of the high prices paid by customers. [3]

Interestingly, even though generous refunds appear to be more attractive to customers, they do not necessarily improve customer welfare. In fact, our analysis suggests that customer welfare is the highest when no refunds are offered, as customizing the refund terms gives the seller additional leverage to extract customers' surplus. In addition, we show that reducing customer valuation

[2] In this scenario, the optimal partial refund has the same effect as the no-refund policy to low-valuation customers, since these customers will not exercise the partial refund.

[3] Another possible explanation is that there is an active secondary market. For example, customers can sell unused non-refundable Disneyland tickets to another person bearing in mind that the tickets cannot be transferred (fractionally) after being used once.

uncertainty does not necessarily benefit the firm. Therefore, the firm may not be interested in reducing customer uncertainty even if it can be done at little or no cost.

The rest of the paper is organized as follows. Section 2 briefly summarizes the most recent and relevant literature. Section 3 presents the model preliminaries and analyzes a benchmark case when refunds are not allowed. Sections 4 and 5 consider designs with standard quality and a standard refund policy, respectively. Section 6 analyzes the fully customized policies with an in-depth discussion of managerial insights. Section 7.1 studies the implications on customer welfare and Section 7.2 investigates the seller's incentive to improve the signal quality. Section 8 concludes. All technical proofs are relegated to the Appendix.

2. Literature

This study connects two streams of literature — consumer refund policy and product line design.

There is an extensive literature with regard to consumer refund policies, covering the various reasons that may trigger consumer returns such as opportunism (Chu et al. 1998, Shang et al. 2017), product quality (Moorthy and Srinivasan 1995, Hsiao and Chen 2012), product mismatch (Davis et al. 1995, Guo 2009, Shulman et al. 2010), alternate options (Xie and Gerstner 2007), unattended service (Gallego et al. 2015), customer no-shows (Ringbom and Shy 2004, Ringbom and Shy 2008) and valuation uncertainty (Che 1996, Courty and Li 2000, Liu and Xiao 2008, Su 2009, Chen 2011, Shulman et al. 2011, Akcay et al. 2013, Akan et al. 2015, Shulman et al. 2015), just to list a few. Furthermore, consumer refund policies have been studied in both manufacturing and service settings. Most of the aforementioned papers focus on the manufacturing setting, with a few dedicated to service settings such as Courty and Li (2000), Xie and Gerstner (2007), Liu and Xiao (2008), Ringbom and Shy (2008), Guo (2009), and Akan et al. (2015). Due to the volume of papers involving consumer refund policies, we do not attempt to conduct a comprehensive literature review. Rather, we will only discuss those papers that are most relevant to ours, namely, papers that study the service settings where returns are triggered by consumers' valuation uncertainty, and those that offer insights on partial refunds.

In the service context, customer refunds are usually triggered by valuation uncertainty, as cancellation normally takes place before a service is experienced. Courty and Li (2000) introduce a price-refund menu as a way for the seller to screen customers of different valuation distributions. Akan et al. (2015) extend the problem by allowing customers to learn their valuations at different times so that the seller can screen on both the amount and timing of the refund. Liu and Xiao (2008) suggest that either the service or the refund should be offered on a more restricted basis when there is a capacity constraint; in addition, they study the impact of capacity rationing on the price-refund menu design. Chen (2011) considers a capacity-constrained seller facing both aggregated demand uncertainty and heterogeneous customer valuations, and develops an optimal selling scheme that induces truthful revelation. Even though we do not consider the capacity issue in the current paper, our results corroborate the idea that refund may not be offered to all customers due to quality customization. We also share a similar interest in the recent literature on the impact of information provision regarding uncertain valuation. Shulman et al. (2015) find that information provision may increase the cancellation rate, particularly when the pre-purchase valuation is moderate with regard to the price. By configuring the optimal quality-price-refund menu, our study further shows that information provision may harm the seller's profit, depending on the level of valuation uncertainty.

Another research question has attracted considerable attention in the literature is when full or partial refunds should be offered. Davis et al (1995) use a stylized model to demonstrate the conditions under which a full refund should be offered. Ringbom and Shy (2004) propose the use of common prices but different partial refunds in order to discriminate among customers of different no-show rates. Xie and Gerstner (2007) show that in the presence of finite capacity, a seller can benefit from a partial refund for service cancellation as freed capacity can be used to serve other consumers. Ringbom and Shy (2008) study collusive pricing and refund policies in the service industry. They show that monopolistic and collusive service providers offer full refund, while competitive service providers offer partial refunds. Guo (2009) studies the rationale behind offering partial refunds in a competitive setting and identifies capacity scarcity as a key driver for partial refunds that are adopted in equilibrium. Su (2009) studies consumer refund policies in a supply chain setting. He shows that it is optimal to offer partial refunds, which are driven by aggregate demand uncertainty. Shulman et al. (2010) also demonstrate the optimality of partial refunds in a supply chain. Their focus is whether the manufacturer or the retailer should process product returns. Shulman et al. (2011) further show that partial refunds can be more frugal in a competitive than monopolistic environment. Hsiao and Chen (2012) illustrate that there are conditions under which refund can exceed the full price. Partial refunds also constitute an important element of our result. We show that partial refunds can emerge as an optimal policy, even when there are no capacity limitations, aggregate demand uncertainty, competition, or channel conflicts. In particular, partial refunds can be offered as a way of screening consumers under a standard refund policy, such that one consumer segment is discouraged from claiming a refund, even if their valuations turn out to be low.

With respect to the stream of research that considers product line design due to valuation uncertainty, the setup of this paper is most relevant to those considered in Guo and Zhang (2012) and Xiong and Chen (2014). Both papers consider a mixture of high- and low-type customers, and study how to design the best product offerings for each type. Specifically, Guo and Zhang (2012) study the impact of consumer deliberation on optimal product design. Xiong and Chen (2014) allow consumers to choose a standard product before learning about their types, or pay for a learning fee and choose afterwards. However, neither of the two papers considers the option of refunds after customers discover their true types.

3. Model Preliminaries

A service provider (henceforth referred to as the "firm," with a masculine pronoun) serves a market with a total demand normalized to 1. The firm can serve the market with one or more products. Each product is defined by a triplet (q, p, β) with quality q, price p, and the refund rate $\beta \in [0,1]$. Products can differ from each other on all or a subset of these attributes. For example, two products can share the same quality but have different prices or refund rates. The cost of serving one unit of product at quality level q is $q^2/2$. If the product is returned, no cost will be incurred and an amount βp will be refunded to the customer. This setup is meant to resonate with service settings, e.g., air travel, Internet services, where cancellation does not incur any materialized cost. However, such a setup may not be suitable for physical goods as returns may involve non-trivial costs such as shipping and repackaging.

3.1 Market Composition

Each customer (henceforth referred to by a feminine pronoun) has a quality valuation θ and derives a product valuation θq for a product with quality q. The valuation parameter θ is heterogeneous among the customers. A fraction α of the customers are high-type with quality valuation θ_H, and the rest $\bar{\alpha} = 1 - \alpha$ are low-type with quality valuation θ_L, where $\theta_L \leq \theta_H$. In order to simplify our notation, we use $\bar{y}$ to denote $1 - y$ for any $y \in [0,1]$ for the rest of the paper.

We assume that the demographic composition, α, is public knowledge. However, customers do not observe their own valuation types prior to purchase. Instead, each customer receives a private signal, "Good" or "Bad," indicating her true type. The quality of the signal can be measured by the probability of getting the right signal conditional on one's true type:

$$P(\text{Good} \mid \text{High-type}) = P(\text{Bad} \mid \text{Low-type}) = \rho,$$

where we assume $\rho \in (1/2, 1]$; thus the signals are informative. Subsequently, we refer to customers receiving a "Good" or "Bad" signal as the "good-signal" or "bad-signal" customers, respectively.

Table 1 Customer categorization based on true valuation and the received signal.

	Good Signal	Bad Signal
High Valuation	$\rho\alpha$	$\bar{\rho}\alpha$
Low Valuation	$\bar{\rho}\bar{\alpha}$	$\rho\bar{\alpha}$

Then, depending on their true valuations and the received signals, customers can be divided into four categories as illustrated in Table 1.

Note that the fraction of good-signal customers is $\rho_G = \rho\alpha + \bar{\rho}\bar{\alpha}$ and the rest $\rho_B = 1 - \rho_G = \rho\bar{\alpha} + \bar{\rho}\alpha$ consists of bad-signal customers. Following Bayes' Theorem, a good-signal customer is of high-type with probability α_G and a bad-signal customer is of high-type with probability α_B, where

$$\alpha_G = \frac{\rho\alpha}{\rho\alpha + \bar{\rho}\bar{\alpha}}, \quad \alpha_B = \frac{\bar{\rho}\alpha}{\bar{\rho}\alpha + \rho\bar{\alpha}}.$$

The expected quality valuations for good- and bad-signal customers are therefore $\theta_G = \alpha_G\theta_H + (1-\alpha_G)\theta_L$ and $\theta_B = \alpha_B\theta_H + (1-\alpha_B)\theta_L$, respectively. Observe that, since $\rho \in (1/2, 1]$, a good signal implies a higher chance of being a high-type customer, as well as higher expected quality valuations, i.e., $\alpha_G > \alpha > \alpha_B$ and $\theta_G > \theta_B$.

3.2 Customer Choice and Market Outcomes

Given that the signals are private for individual customers, the firm faces an adverse selection problem. The firm needs to design a product line with at most two products, product "G", (q_G, p_G, β_G), and product "B", (q_B, p_B, β_B), catering to the good- and bad-signal customers respectively. When $(q_G, p_G, \beta_G) = (q_B, p_B, \beta_B)$, the two products are identical and only one product is offered to all customers. The firm can also choose to drop good- or bad-signal customers by setting the price high enough.

Let u_{si} denote the expected surplus for customers receiving signal s and purchasing product i, where $s, i \in \{G, B\}$. To induce the good-signal customers to choose product "G" and bad-signal customers to choose product "B", the firm needs to ensure that both individual rationality (IR) and incentive compatibility (IC) constraints are satisfied. That is,

$$u_{GG} \geq 0, \qquad (IR_G)$$
$$u_{BB} \geq 0, \qquad (IR_B)$$
$$u_{GG} \geq u_{GB}, \qquad (IC_G)$$
$$u_{BB} \geq u_{BG}. \qquad (IC_B)$$

The expected surplus can be characterized by acknowledging the fact that a customer will return the product if and only if the potential refund exceeds the product valuation. Thus $u_{si} = \alpha_s \max\{\theta_H q_i - p_i, -\bar{\beta}_i p_i\} + \bar{\alpha}_s \max\{\theta_L q_i - p_i, -\bar{\beta}_i p_i\}$. Further, it can be shown that only the low-type customers *may* exercise the refund, i.e., $\max\{\theta_H q_i - p_i, -\bar{\beta}_i p_i\} = \theta_H q_i - p_i$ for $i \in \{G, B\}$. Otherwise, both (IR) constraints will be violated. The expected surplus for customers receiving signal s and purchasing product i is then given by

$$u_{si} = \alpha_s(\theta_H q_i - p_i) + \bar{\alpha}_s \max\{\theta_L q_i - p_i, -\bar{\beta}_i p_i\} = \alpha_s \theta_H q_i + \bar{\alpha}_s \max\{\theta_L q_i, \beta_i p_i\} - p_i. \tag{1}$$

Therefore, a customer who has purchased a product $i \in \{G, B\}$ will return the product if and only if (i) she is of low-type ($\theta = \theta_L$), and (ii) the refund exceeds her product valuation ($\theta_L q_i \leq \beta_i p_i$). With respect to the above, there are four possible market outcomes with regard to customer refunds:

RR: all low-type customers will claim the refund;

NR: only bad-signal (purchasing product "B") low-type customers will claim the refund;

RN: only good-signal (purchasing product "G") low-type customers will claim the refund;

NN: no customer will claim the refund.

In order to design the optimal product line, the firm needs to investigate the best product line design under each market outcome, and compare the profits across the four candidate solutions.

3.3 Product Line Design without a Refund

As a benchmark case, we analyze the situation where no refund is allowed, i.e., $\beta_G = \beta_B = 0$. This case also corresponds to the market outcome **NN**. Note that the firm can still choose between a single-product or dual-product design.

Under the single-product design, only one product $(p, q, 0)$ will be offered and the firm needs to decide whether it will serve only good-signal customers or the entire market. The former calls for solving the problem:

$$\max_{p,q \geq 0} \; \rho_G \left(p - q^2/2\right)$$
$$\text{s.t. } \theta_G q \geq p \geq \theta_B q;$$

while the latter invokes the following problem:

$$\max_{p,q \geq 0} \; p - q^2/2$$
$$\text{s.t. } \theta_B q \geq p.$$

The optimal solution corresponds to the maximum between the two.

Under the dual-product design, on the other hand, the firm solves

$$\max_{q_G,q_B,p_G,p_B\geq 0} \quad \rho_G(p_G - q_G^2/2) + \bar{\rho}_G(p_B - q_B^2/2)$$
$$\text{s.t. } (IR_G),(IR_B),(IC_G),(IC_B).$$

The results for the single-product design (denoted with the superscript "NS") and the dual-product design (denoted with the superscript "ND") are summarized in Proposition 1. Before proceeding, we introduce the following notations and relations that will be used in the remainder of the paper:

$$\phi(\lambda,\alpha_1,\alpha_2) = \frac{\lambda\alpha_2 - \alpha_1}{\bar{\alpha}_1 - \lambda\bar{\alpha}_2}, \qquad \forall\, 0 \leq \alpha_1 < \alpha_2 \leq 1, 0 \leq \lambda \leq 1,$$
$$\alpha_0 = \frac{\alpha_B - \rho\alpha}{\bar{\rho}_G} \leq \alpha_B \leq \alpha \leq \alpha_G,$$
$$\theta_0 = \alpha_0\theta_H + \bar{\alpha}_0\theta_L \leq \theta_B \leq \theta_G.$$

Proposition 1 **(Optimal product line design without a refund)**. *Suppose no refund is offered, then*

(i) the optimal single-product design is

$$(q^{NS}, p^{NS}) = \begin{cases} (\theta_G, \theta_G^2), \text{ if } \frac{\theta_L}{\theta_H} \leq \phi^{NS}, \\ (\theta_B, \theta_B^2), \text{ otherwise,} \end{cases}$$

where $\phi^{NS} = \phi(\sqrt{\rho_G}, \alpha_B, \alpha_G) > 0$*;*

(ii) the optimal dual-product design is

$$(q_G^{ND}, p_G^{ND}) = (\theta_G, \theta_G(\theta_G - \theta_0) + \theta_0\theta_B),$$
$$(q_B^{ND}, p_B^{ND}) = (\theta_0, \theta_0\theta_B);$$

(iii) the firm is always better off under the dual-product design.

Proposition 1(i) shows that under the single-product design, it is optimal to serve only the good-signal customers at a relatively high quality level θ_G when the valuation heterogeneity is high (i.e., θ_L/θ_H is low), or the whole market at a relatively low quality level θ_B when the valuation heterogeneity is low (i.e., θ_L/θ_H is high). However, Proposition 1(ii) shows that the valuation heterogeneity is less relevant in the optimal dual-product design. Under the dual-product design, good-signal customers will be served with relatively high quality (θ_G), and bad-signal customers will be served with even lower quality (θ_0) than under the single-product design. Proposition 1(iii) shows that the firm is always better off under the dual-product design, which may not actually be true when refunds are allowed, as shown later in the paper.

4. Optimal Product Line Design with a Single Quality Level

Although our main focus is to study the product line design that maximizes the firm's profit, in this section, we consider the situation where only a single quality level is possible (thereby denoted with the superscript "Q"). This happens, for example, when it is rather costly to provide differentiated service qualities.

The firm then needs to design a single quality level $q = q_G = q_B$ and two products (q, p_G, β_G) and (q, p_B, β_B), targeting good- and bad-signal customers, respectively. As discussed in Section 3.2, in order to find the optimal product line design, the firm compares the optimal product lines under each possible market outcome. As an example, the optimal design with the market outcome **RR** can be identified by solving:

$$\max_{q,p_G,p_B,\beta_G,\beta_B \geq 0} \Pi^{RR} = \rho_G(p_G - \bar{\alpha}_G\beta_G p_G) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta_B p_B) - \frac{\alpha q^2}{2}$$

$$\text{s.t.} \quad \begin{array}{lcll} u_{GG} & \geq & 0, & (IR_G) \\ u_{BB} & \geq & 0, & (IR_B) \\ u_{GG} & \geq & u_{GB}, & (IC_G) \\ u_{BB} & \geq & u_{BG}, & (IC_B) \\ \beta_G p_G & \geq & \theta_L q, & (RR_G) \\ \beta_B p_B & \geq & \theta_L q, & (RR_B) \\ \beta_G & \leq & 1, & \\ \beta_B & \leq & 1. & \end{array}$$

The (IR)'s and (IC)'s constraints ensure that the products are targeting the right set of customers, and the (RR) constraints ensure the **RR** market outcome — that all low-valuation customers, whether purchasing product G or B, will exercise the refund. The firm's profit in the objective function includes revenues, net of refunds collected from all customers, minus the service cost for all high-valuation customers (with size α) who choose to keep the product.

The problem under the market outcome **NN** has already been characterized in Proposition 1(i). The problem formulations for the market outcomes **NR** and **RN** can be written in a similar fashion; we relegate the details to the Appendix. Comparing profits across all market outcomes delivers the following result:

PROPOSITION 2 **(Optimal Design with a Single Quality Level)**. *When only a single quality is offered ($q_G = q_B = q$), there exist $\bar{\phi}^Q \geq \underline{\phi}^Q \geq 0$ with*

$$\underline{\phi}^Q = \min\left\{\max\left\{\phi(\sqrt{\alpha}, \alpha_B, 1), \phi\left(\sqrt{1-\rho\bar{\alpha}}, \alpha_B, \frac{\alpha}{1-\rho\bar{\alpha}}\right)\right\}, \phi\left(\sqrt{\frac{\alpha}{1-\rho\bar{\alpha}}}, \frac{\alpha}{1-\rho\bar{\alpha}}, 1\right)\right\},$$

$$\bar{\phi}^Q = \max\left\{\phi(\sqrt{\alpha}, \alpha_B, 1), \phi\left(\sqrt{1-\rho\bar{\alpha}}, \alpha_B, \frac{\alpha}{1-\rho\bar{\alpha}}\right), \phi\left(\sqrt{\frac{\alpha}{1-\rho\bar{\alpha}}}, \frac{\alpha}{1-\rho\bar{\alpha}}, 1\right)\right\}$$

such that

*(i) if $\frac{\theta_L}{\theta_H} \le \underline{\phi}^Q$, the market outcome is **RR**. The optimal product design is*

$$q^Q = \theta_H,\ p_G^Q \in [\alpha_G \theta_H^2, \theta_H^2],\ \beta_G^Q = \frac{1}{\bar{\alpha}_G} - \frac{\alpha_G \theta_H^2}{\bar{\alpha}_G p_G^Q} \in [0,1],\ (p_B^Q, \beta_B^Q) = (\theta_H^2, 1);$$

*(ii) if $\underline{\phi}^Q \le \frac{\theta_L}{\theta_H} \le \bar{\phi}^Q$, the market outcome is **NR**. The optimal product design is*

$$q^Q = \frac{\alpha\theta_H + \bar{\rho}\bar{\alpha}\theta_L}{1-\rho\bar{\alpha}}, (p_G^Q, \beta_G^Q) = (\theta_G q^Q, 0), (p_B^Q, \beta_B^Q) = (\theta_H q^Q, 1);$$

*(iii) if $\frac{\theta_L}{\theta_H} \ge \bar{\phi}^Q$, the market outcome is **NN**. The optimal product design is characterized in Proposition 1(i).*

As the degree of valuation heterogeneity increases (θ_L/θ_H decreases), the firm offers a more liberal refund policy, together with higher service quality. In particular, when the valuation heterogeneity is high ($\theta_L/\theta_H \le \underline{\phi}^Q$), a firm offers the highest quality possible and allows customers to trade off between price and refund. High valuation heterogeneity can be the result of the consumption state. Consider air travel, for instance, a customer whose trip is interruppted by a schedule conflict will have a low valuation for the trip. Our result here seems to coincide with the practice of business class tickets and conference hotels.

As the valuation heterogeneity becomes moderate ($\underline{\phi}^Q < \theta_L/\theta_H \le \bar{\phi}^Q$), bad-signal customers will always choose the product with a full refund, while good-signal customers will opt for a bargain price without a return option. The moderate valuation heterogeneity aligns well with leisure travellers, whose choices are often driven by preferences among product features rather than exogenous needs. Naturally, customers who are less knowledgeable about a destination hotel may pick a higher rate that allows free cancellation, while those who are better informed of the service are more likely to book at a lower, non-refundable rate. As will be discussed next, this observation may be flipped when a standard refund rate applies across products.

5. Optimal Product Line Design under a Standard Refund Rate

This section studies the situation in which the same refund rate (thereby denoted with the superscript "R"), or a standard refund rate, is offered across products. This situation arises when it is difficult to explain differentiated refund rates to customers, or there are regulations forbidding service providers from discriminating among customers by using cancellation terms.

Under a standard refund rate where $\beta_G = \beta_B = \beta$ for some $\beta \in [0,1]$, the firm designs products (q_G, p_G, β) and (q_B, p_B, β) for good- and bad-signal customers, respectively. Following a similar path of analysis as that in the previous section, the optimal design for the market outcome **RR** can be found via the following problem formulation:

$$\max_{q_G, q_B, p_G, p_B, \beta \ge 0} \Pi^{RR} = \rho_G \left(p_G - \bar{\alpha}_G \beta p_G - \frac{\alpha_G q_G^2}{2} \right) + \bar{\rho}_G \left(p_B - \bar{\alpha}_B \beta p_B - \frac{\alpha_B q_B^2}{2} \right)$$

$$\text{s.t.}\quad \begin{array}{lcll} u_{GG} & \geq & 0, & (IR_G) \\ u_{BB} & \geq & 0, & (IR_B) \\ u_{GG} & \geq & u_{GB}, & (IC_G) \\ u_{BB} & \geq & u_{BG}, & (IC_B) \\ \beta p_G & \geq & \theta_L q_G, & (RR_G) \\ \beta p_B & \geq & \theta_L q_B, & (RR_B) \\ \beta & \leq & 1. & \end{array}$$

Again, the analysis for the market outcome **NN** has already been characterized in Proposition 1(ii) and those for the market outcomes **NR** and **RN** are produced in the Appendix.

PROPOSITION 3 **(Optimal Product Line Design with a Standard Refund Rate)**. *When a standard refund rate is offered ($\beta_G = \beta_B$), there exist $\bar{\phi}^R \geq \underline{\phi}^R \geq 0$ with*

$$\underline{\phi}^R = \min\{\phi(\sqrt{\alpha_G}, \alpha_G, 1), \phi(\sqrt{\alpha_B}, \alpha_0, 1)\},$$
$$\bar{\phi}^R = \phi(\sqrt{\alpha_G}, \alpha_G, 1)$$

such that

*(i) if $\frac{\theta_L}{\theta_H} \leq \underline{\phi}^R$, the market outcome is **RR**. The optimal product line is*

$$(q_G^R, p_G^R) = (q_B^R, p_B^R) = (\theta_H, \theta_H^2), \quad \beta^R = 1;$$

*(ii) if $\underline{\phi}^R \leq \frac{\theta_L}{\theta_H} \leq \bar{\phi}^R$, the market outcome is **RN**. The optimal product line is*

$$(q_G^R, p_G^R) = (\theta_H, \theta_H\theta_G - \theta_0\theta_G + \theta_0\theta_B), \ (q_B^R, p_B^R) = (\theta_0, \theta_0\theta_B), \ \beta^R = \theta_L q_G^R / p_G^R < 1.$$

*(iii) if $\frac{\theta_L}{\theta_H} \geq \bar{\phi}^R$, the market outcome is **NN**. The optimal product design is characterized in Proposition 1(i).*

Similar to the single-quality design analyzed in Section 4, customers enjoy higher service quality and a more liberal refund policy as the valuation heterogeneity increases (θ_L/θ_H decreases) under the standard-refund design. Specifically, when the valuation heterogeneity is high ($\frac{\theta_L}{\theta_H} \leq \underline{\phi}^R$), the standard refund design conforms to a single quality design in that it is optimal to serve all customers at the highest quality level ($q_G^R = q_B^R = q_G^Q = q_B^Q = \theta_G$). However, the standard refund constraint restricts the trade-off between price and refund. Thus the firm has to extend a full refund ($\beta^R = 1$) to all customers. Therefore, although the firm has the flexibility to offer more than one quality level, it will choose to rely on a single product with premium quality and the most liberal refund policy. Such a strategy – limited product variety and high service quality – appears to be adopted in situations with high valuation heterogeneity, such as business flights and conference hotels.

To the contrary of the single-quality design, when the valuation heterogeneity is moderate ($\underline{\phi}^R \leq \frac{\theta_L}{\theta_H} \leq \bar{\phi}^R$), bad-signal customers will take the inferior product and will not claim a refund, while good-signal customers will purchase the high-quality product with a partial refund. The literature has documented various reasons for partial refunds, including aggregate demand uncertainty (Su 2009), capacity limitation (Liu and Xiao 2008), competition (Guo 2009, Ringbom and Shy 2008), and channel conflicts (Shulman et al. 2010). Our model provides evidence that a partial refund can also be driven by valuation heterogeneity and quality customization. In the presence of valuation heterogeneity, the firm needs to distinguish between good- and bad-signal customers. When only a single quality can be offered, the firm relies on highly differentiated refund rates (none vs. full) to fulfill this mandate (Proposition 2(ii)). When quality customization is allowed, however, a common partial refund rate is sufficient to separate the customers (Proposition 3(ii)). Interestingly, the same refund policy triggers different refund decisions across customers — the good-signal customer will exercise the refund, as the refund weakly dominates the valuation of retaining the item ($\beta^R p_G \geq \theta_L q_G$); the bad-signal customers will retain the product, even when their valuations turn out to be low.

6. Optimal Product Line Design with both Quality and Refund Customization

In this section, we analyze the optimal product line design when both the quality and refund rate can be customized. The objective is to develop two products, (q_G, p_G, β_G) and (q_B, p_B, β_B), catering to good- and bad-signal customers in order to maximize the firm's expected profit. In this context, the problem formulation under the market outcome **RR** is given as follows:

$$\max_{q_G, q_B, p_G, p_B, \beta_G, \beta_B \geq 0} \Pi^{RR} = \rho_G \left(p_G - \bar{\alpha}_G \beta_G p_G - \frac{\alpha_G q_G^2}{2} \right) + \bar{\rho}_G \left(p_B - \bar{\alpha}_B \beta_B p_B - \frac{\alpha_B q_B^2}{2} \right)$$

$$\text{s.t.} \quad \begin{array}{lll} u_{GG} & \geq 0, & (IR_G) \\ u_{BB} & \geq 0, & (IR_B) \\ u_{GG} & \geq u_{GB}, & (IC_G) \\ u_{BB} & \geq u_{BG}, & (IC_B) \\ \beta_G p_G & \geq \theta_L q_G, & (RR_G) \\ \beta_B p_B & \geq \theta_L q_B, & (RR_B) \\ \beta_G & \leq 1, & \\ \beta_B & \leq 1. & \end{array}$$

Following a similar line of analysis as before, we compare optimal product lines under each market outcome and obtain the following results:

THEOREM 1 **(Optimal Product Line Design with both Quality and Refund Customization).** *When both the quality and refund rate can be customized, there exists* $\bar{\phi}^* \geq \underline{\phi}^* \geq 0$ *with*

$$\underline{\phi}^* = \min\left\{\phi(\sqrt{\alpha_G}, \alpha_G, 1), \phi(\sqrt{\alpha_B}, \alpha_0, 1)\right\}, \qquad \bar{\phi}^* = \max\left\{\phi(\sqrt{\alpha_G}, \alpha_G, 1), \phi(\sqrt{\alpha_B}, \alpha_0, 1)\right\}$$

such that

*(i) if $\frac{\theta_L}{\theta_H} \leq \underline{\phi}^*$, the market outcome is **RR**. The optimal product line is*

$$q_G^* = \theta_H,\ p_G^* \in [\alpha_G \theta_H^2, \theta_H^2],\ \beta_G^* = \frac{1}{\bar{\alpha}_G} - \frac{\alpha_G}{\bar{\alpha}_G}\frac{\theta_H^2}{p_G^*} \in [0,1],$$
$$(q_B^*, p_B^*, \beta_B^*) = (\theta_H, \theta_H^2, 1);$$

*(ii) if $\underline{\phi}^* \leq \frac{\theta_L}{\theta_H} \leq \bar{\phi}^*$, and ρ is large and α is small, the market outcome is **RN**. The optimal product line is*

$$q_G^* = \theta_H,\ p_G^* \in [\theta_G(\theta_H - \theta_0) + \theta_0\theta_B, \theta_H(\theta_H - \theta_0) + \theta_0\theta_B], \beta_G^* = \frac{1}{\bar{\alpha}_G} - \frac{\alpha_G\theta_H^2 - (\theta_G - \theta_B)\theta_0}{\bar{\alpha}_G p_G^*} \in [0,1],$$
$$(q_B^*, p_B^*, \beta_B^*) = (\theta_0, \theta_0\theta_B, 0);$$

*otherwise, and the market outcome is **NR**. The optimal product line is*

$$(q_G^*, p_G^*, \beta_G^*) = (\theta_G, \theta_G^2, 0), \quad (q_B^*, p_B^*, \beta_B^*) = (\theta_H, \theta_H^2, 1);$$

*(iii) if $\frac{\theta_L}{\theta_H} \geq \bar{\phi}^*$, the market outcome is **NN**. The optimal product design is characterized in Proposition 1(i).*

Similar to the single-quality and the standard-refund design, when the firm has full flexibility to customize both, service qualities and refund rates are generally higher as the valuation heterogeneity increases. One ensuing question is the value of flexibility in the product line design. In other words, would the firm still choose a single quality level or a standard refund rate, even when it has full flexibility to customize both? We answer these questions in the forthcoming subsection.

6.1 When is Single Quality or Standard Refund Optimal?

Although the firm possesses the capability of customizing both the quality and refund, it is often sufficient to customize only one of them. We summarize these scenarios as follows:

COROLLARY 1.

(i) Standard quality maximizes the firm's expected profit when the valuation heterogeneity level is high;

(ii) A standard refund maximizes the firm's expected profit when the valuation heterogeneity level is high or low;

(iii) A standard refund in general maximizes the firm's expected profit when the high-type customers does not form the majority, i.e., $\alpha < 0.5$, and the signal quality ρ is high with $\rho \geq \hat{\rho}(\alpha)$, where $\hat{\rho}(\alpha)$ is weakly increasing in α with $\lim_{\alpha\to 0}\hat{\rho}(\alpha) = 0.5$ and $\hat{\rho}(\alpha) = 1$ for any $\alpha \in [0.5, 1]$.

The first two statements follow immediately from Theorem 1. First, the optimal product line can be sustained by a single quality level ($q_G = q_B = \theta_H$) when the heterogeneity level is high ($\theta_L/\theta_H \leq \underline{\phi}^*$). In other words, the flexibility to customize refund rates for different customer segments allows the firm to satisfy customers with less product variety. This variety reduction effect is akin to the seminal result of Moorthy (1984), who show that the customers' incentive compatibility constraints sometimes cause the firm to combine products for different customer segments. In our benchmark analysis with no refund (Proposition 1), the firm would not combine products for the two customer segments. Therefore, variety reduction is a direct result of the firm's ability to offer different refund terms to different customer segments. On the other hand, the optimal product line can rely upon a standard refund rate ($\beta_G = \beta_B$ =0 or 1), when the valuation heterogeneity is low or high ($\theta_L/\theta_H \geq \bar{\phi}^*$ or $\leq \underline{\phi}^*$).

In certain circumstances, standardization applies through all valuation heterogeneity levels. Specifically, the optimal product line adopts the standard refund when the signal is strong (ρ large) and there are fewer high-valuation customers in the market (α small). In this case, a full or no refund will be offered when the valuation heterogeneity is high or low, respectively, and a common partial refund applies when the valuation heterogeneity is moderate. Corollary 1(iii) explicitly characterizes the boundaries for signal strength (ρ) and population distribution (α) that would support the standard refund. In general, the signal strength needs to be over some threshold $\hat{\rho}(\alpha)$ in order to support a standard refund. Further, the threshold $\hat{\rho}(\alpha)$ increases in α; thus more high-valuation customers would require more accurate signals for the firm to choose a standard refund. Specifically, when almost the entire population is of low-valuation ($\alpha \to 0$), the standard refund shall universally apply, since $\lim_{\alpha\to 0} \hat{\rho}(\alpha) = 0.5$. However, when the market is primarily composed of high-valuation customers ($\alpha \geq 0.5$), the standard refund should not be adopted as $\hat{\rho}(\alpha) = 1$ for all $\alpha \geq 0.5$. We illustrate and further discuss this implication in the next subsection.

6.2 The Impact of Market Structure and Signal Quality

To further explore the impact of market structure and signal quality on the optimal product line, we conduct a numerical study with a wide range of model parameters and illustrate the results in Figure 1.

When the signal is perfect (i.e., $\rho = 1$), customers know their valuations before purchasing. Thus, the firm will not allow any refund and the market outcome is **NN**. This is shown on the right edge of each of the four graphs in Figure 1. Hence, a refund in our model is used to accommodate valuation uncertainty. In what follows, we restrict our discussion to scenarios with an imperfect signal only (i.e., $0.5 \leq \rho < 1$).

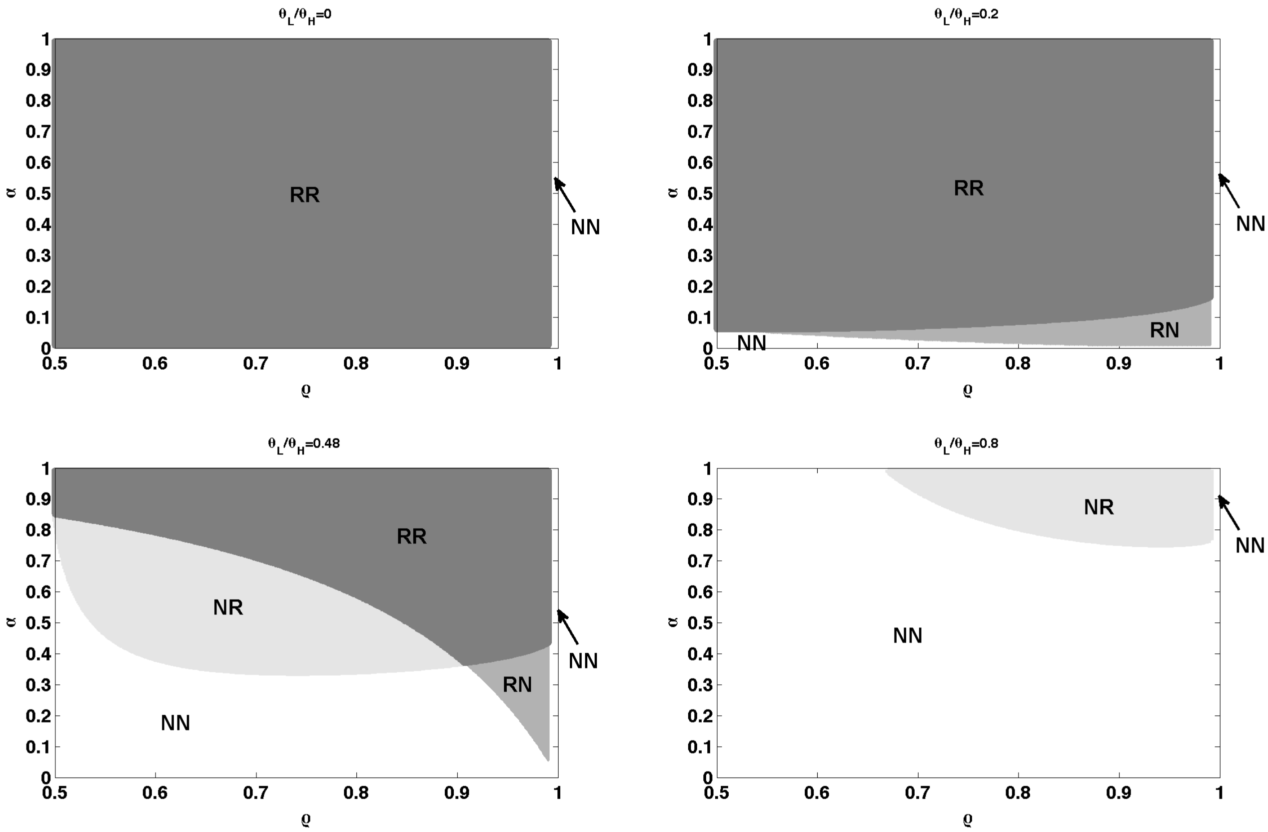

Figure 1 Optimal menu type as the valuation heterogeneity, demography and signal quality vary.

The top left graph describes the scenario where the valuation heterogeneity is high ($\theta_L/\theta_H = 0$) and the low-valuation customer will derive zero utility from the product. In this scenario, the firm will make refunds available to all customers (i.e., the market outcome is **RR**), regardless of the fraction of high-valuation customers (α) or signal quality (ρ).

As the valuation heterogeneity decreases (θ_L/θ_H increases), it is more likely that the refund will be restricted to at least one of the products, but **RR** is still optimal as long as the fraction of high-valuation customers (α) and the valuation heterogeneity are reasonably high. In other words, the boundary conditions for the **RR** outcome ($\underline{\phi}^*$) is characterized by both the fraction α and signal quality ρ. For example, to ensure that $\theta_L/\theta_H = 0.2$ falls below $\underline{\phi}^*$, the fraction α should exceed 0.07 for the null signal ($\rho = 0.5$) and 0.18 for near-perfect signal (ρ approaching 1). For a relatively lower heterogeneity level $\theta_L/\theta_H = 0.48$, α should be over 0.9 (for the null signal) or 0.42 (for the near-perfect signal) for the market outcome to be **RR**.

When the valuation heterogeneity drops below a certain threshold, e.g., $\theta_L/\theta_H > 0.5$, **RR** is never optimal, and **NN** becomes more dominant. In all four graphs, the market outcome **NN** is optimal under conditions that are opposite to those inducive to **RR** — when the fraction of high-valuation customers (α) as well as the valuation heterogeneity are both low. Specifically, when

θ_L/θ_H approaches 1, no refund policy (**NN**) will always be adopted.

When the heterogeneity level is moderate (bounded between $\underline{\phi}^*$ and $\bar{\phi}^*$), the bottom left graph ($\theta_L/\theta_H = 0.48$) illustrates the domain for **RN** and **NR**. In general, **RN** occupies the southeast region while **NR** occupies the northwest. As suggested both analytically and numerically, refund will only be exercised by the good-signal customers (**RN**) if the signal quality is high, but high-valuation customers are rare, and only by bad-signal customers (**NR**) when the signal is of low quality but high-valuation customers are plentiful. The actual market outcome depends upon the valuation heterogeneity (θ_L/θ_H). For example, consider the case with $\alpha = 0.7$ and $\rho = 0.6$, the optimal product line would yield the market outcome **RR** $\rightarrow$ **NR** $\rightarrow$ **NN** as θ_L/θ_H increases from 0 to 1; but with a lower $\alpha = 0.3$ and a more accurate signal $\rho = 0.95$, the market outcome would vary from **RR** $\rightarrow$ **RN** $\rightarrow$ **NN** as θ_L/θ_H increases from 0 to 1.

6.3 Managerial Insights

Our analysis confirms that a wide range of quality levels and refund policies can be optimal. The optimal product line depends on the valuation heterogeneity (θ_L/θ_H), signal quality (ρ), and the demographic composition (fraction of high-valuation customers α). We next discuss the implications of these parameters on industry applications and summarize the insights in Table 2.

The valuation heterogeneity reflects valuation uncertainty on the *aggregated* level, which can arises from several different sources. We label them as *need*, *preference*, and *commodity*. A high heterogeneity level (low θ_L/θ_H) often corresponds to consumption state uncertainty, where customers are either in *need* or *no need* of the service, e.g., business trips and conference hotels. Distinctive from *need*, when the valuation heterogeneity is low (high θ_L/θ_H), all customers share a similar valuation, which is prototypical for *commodities*. Examples include popular concerts/event tickets, deep discount travel products, etc. Finally, a moderate valuation heterogeneity level (intermediate θ_L/θ_H) can arise due to the taste differences among customers, which are commonly referred to as consumer *preference*. This scenario is more complicated than *need* or *commodity*, given that it also requires the valuation uncertainty on the *individual* level — namely, the signal quality (ρ), in determining the optimal product line. Intuitively, a customer is *better*- (resp. *less*-) informed of her valuation type if the signal quality ρ is high (resp. low).

When the valuation heterogeneity is high, the firm will make refunds available to all customers (**RR**) and will offer a single high-quality service to all customers ($q_G = q_B = \theta_H$). In addition, even though it is possible for the firm to offers a price-refund tradeoff to customers at a single quality level, it is sufficient to use a standard quality product (θ_H) and a standard refund policy ($\beta = 1$) to maximize its profit.

Table 2 Implications of the Optimal Product Line Design

Characteristics of Valuation Uncertainty	**Optimal Product Line** (G for good-signal customers and B for bad-signal customers)	**Application Areas**	**Allow Standard-** Quality	Refund
High on the *Aggregate* Level	G: high quality, partial/full refund B: high quality, full refund	***Need-driven markets*** Business-class airfares, conference hotels	Yes	Yes
Moderate on the *Aggregate* Level; **High** on the *Individual* Level	G: good quality, no refund B: high quality, full refund	***Preference-driven, less-informed markets*** Leisure travel, rental markets	No	No
Moderate on the *Aggregate* Level; **Low** on the *Individual* Level	G: high quality, partial refund B: low quality, no refund	***Preference-driven, more-informed markets*** Wireless, Internet service, subscription vs. pay-to-go	No	Yes
Low on the *Aggregate* Level	G: good quality, no refund B: low quality, no refund	***Commodity-like markets*** Concerts/events, theme parks, last-minute booking	No	Yes

When the valuation heterogeneity is low, it is optimal for the firm not to offer any refunds (**NN**), but to discriminate among customers via quality differentiation. Specifically, the firm would design a product line that provides the good-signal customers with a good quality product ($q_G = \theta_G$) and the bad-signal customers with an inferior quality one ($q_B = \theta_0$); neither allows any refund ($\beta_G = \beta_B = 0$). This corresponds with the concert tickets that are offered in many different zones; therefore, there are substantially different qualities, but a common non-refundable policy.

When the valuation heterogeneity is intermediate, the optimal product line is distinct between *less-informed* and *better-informed* markets, the boundary of which is determined by the demographic composition α. The need for better valuation knowledge is higher when the fraction of high-valuation customers is high.

In a *better-informed* market (high ρ), customers are confident about what they need and to what extent they will need it. This knowledge is not as valuable when only a small fraction of customers have high valuations for the product. Consider the Internet phone service, for instance. A customer may routinely use the service up to, say, 100 minutes per month; hence, any additional allowance beyond 100 minutes means little to the customers. In addition, perhaps only a small segment of users, e.g., those who run home businesses, may have a strong demand for the "unlimited" service. According to our findings, it is best for the firm to offer a non-refundable low-cost service (e.g., prepaid or pay-per-use plan) to the mass customers, and a partially refundable, premium service (e.g., an unlimited, all-inclusive plan that can be cancelled, subject to certain conditions) to the niche customers. In practice, the (partial) refund terms for premium services can take different forms, such as early termination fees (AT&T, Bell), a non-refundable monthly unused portion (Skype, Rogers), a termination charge as a fraction of the remaining contract (ADT), and administrative charges (Ringcentral). As implied by our results, prepaid or pay-per-use customers are often not interested in claiming a refund, even thought the same refund terms are offered to them. As a result, the firm can safely rely upon a standard (and partial) refund policy and customized quality to maximize its profit.

Examples of *less-informed* markets include leisure travel and real estate rental, where personal comfort and pleasure constitutes the main component of the perceived service quality, and the majority of customers would put high valuation on premium service. In the meantime, customers' knowledge regarding their own needs is only vaguely understood. For example, it is highly likely that a customer who thought that she could settle with a basic service may later have a strong desire for an upgrade, or vice-versa. Table 2 suggests that, in this scenario, the firm should offer bad-signal customers the highest level of quality with a full refund option, and a good quality product with no refund for good-signal customers. The rationale is that the firm should attract hesitant customers due to negative signals (e.g., leisure travellers who are unfamiliar with a neighbourhood, tenants with ever-changing work locations) by offering a premium quality service and allowing customers to opt out anytime at no cost (e.g., cancellable hotel booking with a breakfast package, a fully furnished rental in flexible terms). At the same time, the firm should simply provide those confident customers (due to a positive signal) with good service and no refund (e.g., a standard hotel room with a discounted but non-refundable rate, an unfurnished apartment that requires a long term commitment, etc). This is notably the only scenario amongst all that requires customization on both the quality and refund policy.

Throughout the four scenarios, the standard refund policy applies except when the valuation uncertainty is moderate on the *aggregate* level but high on the *individual* level, i.e., a *preference*-driven, *less-informed* market. This should be contrasted with the case when valuation uncertainty is high on the *aggregated* level (i.e., a *need* driven market), where the standard quality design is applicable. Therefore, our results suggest that the firm can safely rely on a single-quality design when the valuation heterogeneity is high, or a standard refund rate when the signal quality is strong.

7. Welfare Implications and Information Provision

This section investigates the welfare implications of the optimal product line. Furthermore, we look at whether the firm may have the incentive to influence the signal quality that customers receive.

7.1 Consumer Welfare

How does customer welfare vary across customers? Would customers be better off under customized refund rates than a standard refund rate? We summarize customers' surplus in Tables 3 and 4. Overall, customers fall into four categories depending on their received signals and true valuation types, denoted by **GH**, **GL**, **BH** and **BL**, where the first letter represents the signal ("Good" or "Bad") and the second reflects the valuation type ("High" or "Low"). The surplus for each customer category is calculated under all possible market outcomes induced by the optimal design with a standard (Proposition 3), as well as customized (Theorem 1) refund policy.

Table 3 Consumer welfare under *standard* refund policy

Customer Type	**Full Refund**	**Partial Refund**	**No Refund**
GH	0	$\theta_H(\theta_H - \theta_G) + \theta_0(\theta_G - \theta_B)$	$\theta_G(\theta_H - \theta_G) + \theta_0(\theta_G - \theta_B)$
GL	0	$-\theta_H(\theta_G - \theta_L) + \theta_0(\theta_G - \theta_B) < 0$	$-\theta_G(\theta_G - \theta_L) + \theta_0(\theta_G - \theta_B) < 0$
BH	0	$\theta_0(\theta_H - \theta_B)$	$\theta_0(\theta_H - \theta_B)$
BL	0	$-\theta_0(\theta_B - \theta_L) < 0$	$-\theta_0(\theta_B - \theta_L) < 0$

Table 4 Consumer welfare under *customized* refund policy (when not coinciding with the standard policy)

Customer Type	**Full Refund**	**Customized Refund**	**No Refund**
GH	0	$\theta_G(\theta_H - \theta_G)$	$\theta_G(\theta_H - \theta_G) + \theta_0(\theta_G - \theta_B)$
GL	0	$-\theta_G(\theta_G - \theta_L) < 0$	$-\theta_G(\theta_G - \theta_L) + \theta_0(\theta_G - \theta_B) < 0$
BH	0	0	$\theta_0(\theta_H - \theta_B)$
BL	0	0	$-\theta_0(\theta_B - \theta_L) < 0$

In general, customers who are offered a full refund will always end up with a zero surplus. Other than that, high-valuation customers receive a positive surplus, while low-valuation customers receive a negative surplus. *Ex ante*, good-signal customers expect a positive surplus $\theta_0(\theta_G - \theta_B)$, while bad-signal customers expect a zero surplus, and the same expectation applies to both standard and customized refund policies. This suggests that customizing the refund policy does not necessarily change one's *ex ante* expected surplus.

Note that the optimal product lines under the standard and customized refund may coincide with each other, e.g., when the conditions in Corollary 1(iii) hold. To make the comparison more effective, we consider scenarios where the alternate conditions hold. In this case, it can be verified that

$\underline{\phi}^R = \bar{\phi}^R = \underline{\phi}^* < \bar{\phi}^*$ and the two policies are strictly distinct only when the valuation heterogeneity is within $[\underline{\phi}^*, \bar{\phi}^*]$. In this region, a standard refund will allow *no* refund while the customized refund will offer a *customized* partial refund. Comparing Table 3 and 4, it appears that high-valuation customers are always better off under the standard refund, while low-valuation customers are better off under the customized refund.

These findings are summarized in the following proposition:

PROPOSITION 4. *(i) The refund policy does not change the expected customer surplus based on signals.*

(ii) High-type customers receives the more (resp. less) surplus under the standard (resp. customized) refund policy, while the reverse applies to low-type customers.

(iii) The standard refund increases the variance in customer surplus, while the customized refund reduces the variance in the customer surplus.

7.2 Signal Quality

Given the impact of the signal quality ρ on the optimal product line design, we would like to investigate whether the firm will benefit from improving the signal quality, e.g., by helping customers figure out their true types via enhanced information provision, or by incentivizing customers to do more research about their needs prior to purchasing.

To begin with, we identify the optimal expected profit of the firm under each market outcome:

$$\begin{aligned}
\Pi^{RR} &= \frac{\alpha\theta_H^2}{2} \\
\Pi^{NR} &= \frac{\rho_G\theta_G^2}{2} + \frac{\bar{\rho}_G\alpha_B\theta_H^2}{2} \\
\Pi^{RN} &= \frac{\rho_G\alpha_G\theta_H^2}{2} + \frac{\bar{\rho}_G\theta_0^2}{2} \\
\Pi^{NN} &= \frac{\rho_G\theta_G^2}{2} + \frac{\bar{\rho}_G\theta_0^2}{2}.
\end{aligned}$$

It can be observed that under the market outcome **RR**, where all low-type customers will obtain the refund, the signal quality matters little to the expected profit; other than that, the signal quality affects the firm's profit in different ways. It can be generally established that:

PROPOSITION 5.

(i) The firm has little incentive to improve the signal quality when the valuation heterogeneity level is high;

(ii) The firm may have an incentive to improve the signal quality when it is already high;

(iii) The firm may have an incentive to dilute information and reduce the signal quality when it is already low.

To illustrate the findings in greater detail, we adopt a similar set of parameters as in Figure 1 and plot the firm's optimal profit as a function of the signal quality ρ in Figure 2.

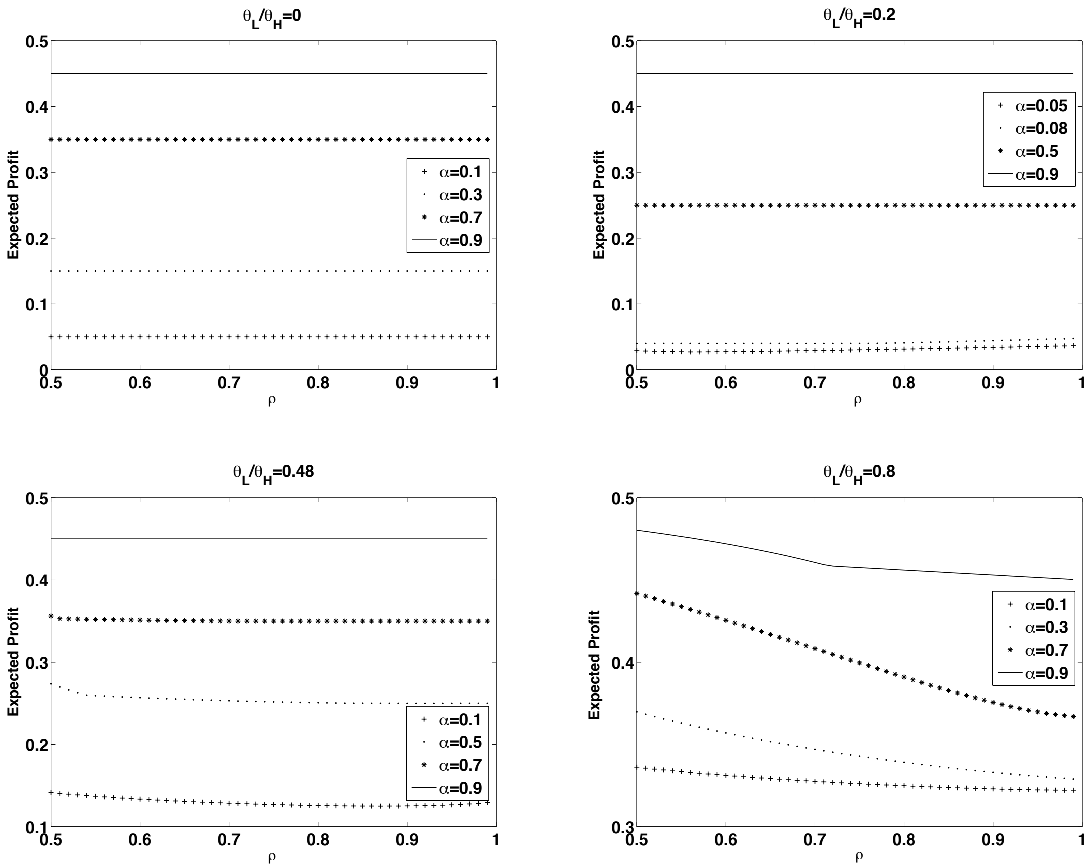

Figure 2 Optimal expected profit as the customer valuation, composition and signal quality vary.

The top graphs show that when the valuation heterogeneity is high, e.g., θ_L/θ_H=0 or 0.2, the firm's profit is generally insensitive to the signal quality due to the dominance of **RR**. For the rare scenarios where the ratio of high-valuation customers is rather low and **RN** dominates (e.g., $\alpha = 0.05$ and 0.08 when $\theta_L/\theta_H = 0.2$), signal quality improvement can somewhat benefit the firm when it is already high (e.g., ρ greater than 0.7).

When the valuation heterogeneity is moderate, e.g., θ_L/θ_H=0.48, improving the signal quality may further reduce the firm's profit, particularly if the signal is weak to begin with. For example, when $\alpha = 0.7$, the expected profit decreases with ρ in $[0.5, 0.77]$ and for $\alpha = 0.5$ the decreasing range extends to $[0.5, 0.85]$. In both instances the market outcome shifts from **NN** to **NR**. For a low fraction of high-valuation customers, e.g., $\alpha = 0.1$, the **NN** menu applies to most of the areas and the expected profit function is convex, decreasing for $\rho \in [0.5, 0.88]$ and increasing for $\rho \in [0.88, 1]$.

For high valuation heterogeneity as in the last graph where $\theta_L/\theta_H = 0.8$, as shown in the bottom right graph, the firm's expected profit will decrease in signal quality.

In connection with the notions in Table 2, we summarize the managerial implications of the above findings in Table 5. When the valuation uncertainty is dominated by *need*, the firm may not be as much motivated to provide further information. If the valuation uncertainty is due to *preference*, the firm may wish to magnify the current signal style – that is, strengthen the already strong signal and weaken any ambiguous signal. Lastly, when the valuation uncertainty is mainly driven by general market conception described as *commodity*, the firm may have a strong incentive to weaken any existing signal, e.g., providing complicated and unorganized information that makes it difficult for customers to understand their degree of valuation.

Table 5 Firm's Information Provision Strategy

Sources of Valuation Uncertainty	**Need**	**Preference**	**Commodity**
Firm's Information Provision Strategy	Take no action	Strengthen strong signal, Weaken ambiguous signal	Weaken all signals

8. Summary and Future Directions

In this paper, we investigate the optimal product line design with consumer refund policies. In our model, returns occur due to valuation uncertainty, reflected by the valuation heterogeneity on the *aggregate* level, and the quality of the signals that customers receive before purchasing at the *individual* level. We study how the valuation heterogeneity and signal quality may affect the quality and refund options available to customers. In general, a more liberal (resp. restricted) refund policy is offered if the valuation heterogeneity is high (resp. low). When the valuation heterogeneity is moderate, the signal quality is the key driver as to which customer segment will get the refund. Furthermore, the firm can safely rely on a single-quality design when the valuation heterogeneity is high, or a standard refund rate when the signal quality is strong. The findings explain various real-life observations, and serve as a helpful tool for service regarding future product line designs.

There are several meaningful dimensions in which the paper can be extended. First, we have focused on the service setting and have ignored the impact of the salvage value and transaction cost. The assumption is reasonable when costs are not incurred until service delivery, and that transaction cost can be negligible due to the increasing popularity of processing customer requests online. However, it is still valuable to examine the impact of the salvage value and transaction costs on an optimal product line and refund policy design. Our model can be extended to incorporate these factors. Such an extension can be applied to the manufacturing setting, where returned products may yield a salvage value or may incur repackaging costs. In this context, the product line design, refund policy, and inventory management needs to be jointly managed together.

Second, given the perishability of service capacity, it is promising to consider the problem under a multi-period capacitated setting and study to the extent to which the perishability and capacity availability may affect the optimal product line. Although the capacity issue has not been the central focus for most marketing papers, its relevancy has been shown to be critical in some recent literature (Xie and Gerstner 2007, Liu and Xiao 2008, Guo 2009). From an operational perspective, it is non-trivial to study how the optimal product line and refund policy design should be adapted to different capacitated environments.

Finally, while our results are derived under a monopolistic setting, it would also be interesting to investigate them in a competitive environment. The effect of competition has been studied in several recent papers (e.g., Guo 2009, Shulman et al. 2011). However, to our knowledge, a competitive product line design with consumer refunds has not been considered so far. This opens another door to interested researchers for future exploration.

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Appendix

We use the following lemmas throughout our proofs. The lemmas can be shown after some algebra. We state them without proof.

LEMMA A1. *Let $0 \le \alpha_1 < \alpha_2 \le 1$ and $0 \le \lambda \le 1$. The following statements hold:*

(i) $\frac{\mathsf{E}_{\alpha_1}[\theta]}{\mathsf{E}_{\alpha_2}[\theta]} \ge (\le) \lambda$ *is equivalent to*

$$\frac{\theta_L}{\theta_H} \ge (\le) \phi(\lambda, \alpha_1, \alpha_2) = \frac{\lambda\alpha_2 - \alpha_1}{\bar{\alpha}_1 - \lambda\bar{\alpha}_2};$$

(ii) $\phi(\lambda, \alpha_1, \alpha_2)$ *decreases with* α_1*, and increases with* α_2 *and* λ*.*

(iii) $\phi(\sqrt{\alpha_1}, \alpha_1, 1) = \frac{\sqrt{\alpha_1}}{1+\sqrt{\alpha_1}}$ *increases with* α_1*.*

(iv) $\phi(\sqrt{\frac{\alpha_1}{\alpha_2}}, \alpha_1, \alpha_2) = \frac{\sqrt{\alpha_1}(\sqrt{\alpha_2}-\sqrt{\alpha_1})}{1+\sqrt{\alpha_1/\alpha_2}+\sqrt{\alpha_1}(\sqrt{\alpha_2}-\sqrt{\alpha_1})} \le \phi(\sqrt{\alpha_1}, \alpha_1, 1)$ *for* $0 \le \alpha_1 < \alpha_2 \le 1$*.*

LEMMA A2. *For* $\rho \ge (\frac{1}{2}, 1]$*, we have*

$$\alpha_B \le \alpha \le \alpha_G, \tag{A1}$$

$$\bar{\alpha}_G \le \bar{\alpha} \le \bar{\alpha}_B. \tag{A2}$$

Proof of Proposition 1

(i) For single-product design, when only good-signal customers are served, the optimal design is $q^* = \theta_G$ and $p^* = \theta_G^2$, with total profit $\rho_G\theta_G^2/2$. When all customers are served, the optimal design is $q^* = \theta_B$ and $p^* = \theta_B^2$, with total profit $\theta_B^2/2$. The former gives higher profit if $\theta_B^2/\theta_G^2 \le \rho_G$ and lower profit otherwise. By Lemma A1, the optimal profit is

$$\begin{aligned}\Pi^* &= \max\{\frac{\rho_G\theta_G^2}{2}, \frac{\theta_B^2}{2}\} \\ &= \begin{cases} \frac{\rho_G\theta_G^2}{2} & \text{if } \frac{\theta_L}{\theta_H} \le \phi(\sqrt{\rho_G}, \alpha_B, \alpha_G) \\ \frac{\theta_B^2}{2} & \text{if } \frac{\theta_L}{\theta_H} \ge \phi(\sqrt{\rho_G}, \alpha_B, \alpha_G) \end{cases}\end{aligned} \tag{A3}$$

(ii) For the dual-product design problem, note that

$$\theta_G q_G - p_G \ge \theta_G q_B - p_B \ge \theta_B q_B - p_B \ge (\theta_B q_G - p_G)^+.$$

Since the objective function is strictly increasing in p_G, at optimality, we must have

$$\theta_G q_G^* - p_G^* = \theta_G q_B^* - p_B^*, \tag{A4}$$

which suggests that $p_G^* - p_B^* = \theta_G(q_G^* - q_B^*)$. Furthermore, the constraint $\theta_B q_B - p_B \geq \theta_B q_G - p_G$ suggests that $p_G^* - p_B^* \geq \theta_B(q_G^* - q_B^*)$. Since $\theta_G \geq \theta_B$, at optimality $p_G^* \geq p_B^*$ and $q_G^* \geq q_B^*$. Thus, we can simultaneously increase p_G and p_B to maintain (A4). It follows that at optimality

$$\theta_B^* q_B^* - p_B^* = 0. \tag{A5}$$

The optimization problem can be rewritten as

$$\max_{q_G, q_B \geq 0} \rho_G(\theta_G q_G - \theta_G q_B + \theta_B q_B - q_G^2/2) + \bar{\rho}_G(\theta_B q_B - q_B^2/2).$$

The optimal quality levels are then

$$q_G^* = \theta_G,\ \ q_B^* = \theta_G - \frac{\theta_G - \theta_B}{\bar{\rho}_G} = \theta_0. \tag{A6}$$

The respective optimal prices are given by

$$\begin{aligned}
p_B^* &= \theta_B q_B^* = \theta_G\theta_B - \frac{(\theta_G - \theta_B)\theta_B}{\bar{\rho}_G} = \theta_0\theta_B, \\
p_G^* &= p_B^* + \theta_G(q_G^* - q_B^*) = \theta_G\theta_B + \frac{(\theta_G - \theta_B)^2}{\bar{\rho}_G} = \theta_G(\theta_G - \theta_0) + \theta_0\theta_B.
\end{aligned}$$

(iii) The optimal profit under dual-product design is

$$\Pi^* = \rho_G\frac{\theta_G^2}{2} + \bar{\rho}_G\frac{\theta_0^2}{2} = \rho_G\frac{\theta_G^2}{2} + \frac{(\theta_B - \rho_G\theta_G)^2}{2\bar{\rho}_G} = \frac{\theta_B^2}{2} + \frac{\rho_G}{2\bar{\rho}_G}(\theta_G - \theta_B)^2 \tag{A7}$$

which is greater than the optimal profit under single-product design $\max\{\theta_B^2/2, \rho_G\theta_G^2/2\}$. Hence the firm is better off under dual-product design than under single-product design. ∎

Proof of Proposition 2

- **RR.** If the menus are of RR, the problem for the firm can be formulated as follows

$$\textbf{RR}: \quad \max_{p_G, p_B, \beta_G, \beta_B \geq 0} \Pi^{RR} = \rho_G(p_G - \bar{\alpha}_G\beta_G p_G) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta_B p_B) - \alpha\frac{q^2}{2}$$

$$\text{s.t.} \quad \begin{array}{lcll}
u_{GG} & \geq & 0 & (IR_G) \\
u_{BB} & \geq & 0 & (IR_B) \\
u_{GG} & \geq & u_{GB} & (IC_G) \\
u_{BB} & \geq & u_{BG} & (IC_B) \\
\beta_G p_G & \geq & \theta_L q & (RR_G) \\
\beta_B p_B & \geq & \theta_L q & (RR_B) \\
\beta_G & \leq & 1 & \\
\beta_B & \leq & 1 &
\end{array}$$

All good-signal customers will choose (p_G, β_G) and pay p_G upfront. However, a fraction $\bar{\alpha}_G$ of the good-signal customers will later find their type to be L and return the product, which costs the firm $\beta_G p_G$. The second term of the objective function can be explained similarly.

The problem then becomes

$$\begin{array}{lll} \textbf{RR}: & \max_{p_G,p_B,\beta_G,\beta_B} & \rho_G(p_G-\bar{\alpha}_G\beta_G p_G)+\bar{\rho}_G(p_B-\bar{\alpha}_B\beta_B p_B)-\alpha\frac{q^2}{2} \\ & \text{s.t.} & \begin{array}{lcll} p_G-\bar{\alpha}_G\beta_G p_G & \le & \alpha_G\theta_H q & (IR_G) \\ p_B-\bar{\alpha}_B\beta_B p_B & \le & \alpha_B\theta_H q & (IR_B) \\ p_G-\bar{\alpha}_G\beta_G p_G & \le & p_B-\bar{\alpha}_G\beta_B p_B & (IC_G) \\ p_B-\bar{\alpha}_B\beta_B p_B & \le & p_G-\bar{\alpha}_B\beta_G p_G & (IC_B) \\ \beta_G p_G & \ge & \theta_L q & (RR_G) \\ \beta_B p_B & \ge & \theta_L q & (RR_B) \\ \beta_G & \le & 1 & \\ \beta_B & \le & 1 & \end{array} \end{array}$$

The two IC constraints can be written as

$$\bar{\alpha}_B(\beta_G p_G-\beta_B p_B)\le p_G-p_B\le\bar{\alpha}_G(\beta_G p_G-\beta_B p_B).$$

Recall that $\rho\in(1/2,1]$, implying $\alpha_G>\alpha>\alpha_B$. Hence, we must have $p_G\le p_B$ and $\beta_G p_G\le\beta_B p_B$. Furthermore, the objective function is increasing in the left hand sides of the two IR constraints, implying the two IR constraints should be binding in optimum. We also comment that (IR_G) and (RR_G) implies that $\beta_G\ge\frac{\theta_L}{\alpha_G\theta_H+\bar{\alpha}_G\theta_L}$. Similarly, (IR_B) and (RR_B) implies that $\beta_B\ge\frac{\theta_L}{\alpha_B\theta_H+\bar{\alpha}_B\theta_L}$.

The optimum can be achieved by any of the following solution:

$$p_G^*\in[\alpha_G\theta_H q,\theta_H q],\ \beta_G^*=\frac{1}{\bar{\alpha}_G}-\frac{\alpha_G}{\bar{\alpha}_G}\frac{\theta_H q}{p_G},\ p_B^*=\theta_H q,\ \beta_B^*=1 \tag{A8a}$$

and

$$\Pi^*=\alpha\theta_H q-\alpha\frac{q^2}{2}.$$

The optimal quality is $q^{RR}=\theta_H$ and $\Pi^{RR}=\frac{\alpha\theta_H^2}{2}$. In particular, the solutions include the following price-return pairs:

$$\begin{array}{l} p_G^{RR}=\theta_H\theta_G,\ \beta_G^{RR}=\theta_L/\theta_G,\ p_B^{RR}=\frac{\theta_H^2}{2},\ \beta_B^{RR}=1,\ \ or \\ p_G^{RR}=\theta_H^2,\ \beta_G^{RR}=1,\ p_B^{RR}=\frac{\theta_H^2}{2},\ \beta_B^{RR}=1. \end{array}$$

- **NR.** In the case when refund only goes to bad-signal customers, the refund rate for menu G can simply be set at 0. The problem can be formulated as follows

$$\begin{array}{lll} \textbf{NR}: & \max_{p_G,p_B,\beta_B\ge 0} & \Pi^{NR}=\rho_G p_G+\bar{\rho}_G(p_B-\bar{\alpha}_B\beta_B p_B)-(1-\rho\bar{\alpha})\frac{q^2}{2} \\ & \text{s.t.} & \begin{array}{lcll} u_{GG} & \ge & 0 & (IR_G) \\ u_{BB} & \ge & 0 & (IR_B) \\ u_{GG} & \ge & u_{GB} & (IC_G) \\ u_{BB} & \ge & u_{BG} & (IC_B) \\ \beta_B p_B & \ge & \theta_L q & (NR_L) \\ \beta_B & \le & 1 & \end{array} \end{array}$$

Or equivalently,

$$\textbf{NR}: \quad \max_{p_G,p_B,\beta_B\geq 0} \quad \rho_G p_G + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta_B p_B) - (1-\rho\bar{\alpha})\frac{q^2}{2}$$
$$\text{s.t.} \quad \begin{array}{lcll} p_G & \leq & \theta_G q & (IR_G) \\ p_B - \bar{\alpha}_B\beta_B p_B & \leq & \alpha_B\theta_H q & (IR_B) \\ p_G - \bar{\alpha}_G\theta_L q & \leq & p_B - \bar{\alpha}_G\beta_B p_B & (IC_G) \\ p_B - \bar{\alpha}_B\beta_B p_B & \leq & p_G - \bar{\alpha}_B\theta_L q & (IC_B) \\ \beta_B p_B & \geq & \theta_L q & (NR_B) \\ \beta_B & \leq & 1 & \end{array}$$

Followed by a similar analysis as in RR, the optimum can be obtained as follows:

$$p_G^* = \theta_G q, \;\; \beta_G^* = 0, \;\; p_B^* = \theta_H q, \;\; \beta_B^* = 1. \tag{A10a}$$
$$\Pi^* = \alpha\theta_H q + \bar{\rho}\bar{\alpha}\theta_L q - (1-\rho\bar{\alpha})\frac{q^2}{2} \tag{A10b}$$

Thus $q^{NR} = \frac{\alpha\theta_H + \bar{\rho}\bar{\alpha}\theta_L}{1-\rho\bar{\alpha}}$ and $\Pi^{NR} = \frac{(\alpha\theta_H + \bar{\rho}\bar{\alpha}\theta_L)^2}{2(1-\rho\bar{\alpha})}$.

Obviously, $\Pi^{NR} \geq \Pi^{RR}$.

• **RN.** When the refund only goes to good-signal customers, i.e., $\beta_B = 0$, the following problem needs to be solved:

$$\textbf{RN}: \quad \max_{p_G,p_B,\beta_G\geq 0} \Pi^{RN} = \rho_G(p_G - \bar{\alpha}_G\beta_G p_G) + \bar{\rho}_G p_B - (1-\bar{\rho}\bar{\alpha})\frac{q^2}{2}$$
$$\text{s.t.} \quad \begin{array}{lcll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & u_{GB} & (IC_G) \\ u_{BB} & \geq & u_{BG} & (IC_B) \\ \beta_G p_G & \geq & \theta_L q & (RN_G) \\ \beta_G & \leq & 1 & \end{array}$$

which becomes

$$\textbf{RN}: \quad \max_{p_G,p_B,\beta_G\geq 0} \quad \rho_G(p_G - \bar{\alpha}_G\beta_G p_G) + \bar{\rho}_G p_B - (1-\bar{\rho}\bar{\alpha})\frac{q^2}{2}$$
$$\text{s.t.} \quad \begin{array}{lcll} p_G - \bar{\alpha}_G\beta_G p_G & \leq & \alpha_G\theta_H q & (IR_G) \\ p_B & \leq & \theta_B q & (IR_B) \\ p_G - \bar{\alpha}_G\beta_G p_G & \leq & p_B - \bar{\alpha}_G\theta_L q & (IC_G) \\ p_B - \bar{\alpha}_B\theta_L q & \leq & p_G - \bar{\alpha}_B\beta_G p_G & (IC_B) \\ \beta_G p_G & \geq & \theta_L q & (RN_H) \\ \beta_G & \leq & 1 & \end{array}$$

The two IC's imply that $\bar{\alpha}_B(\beta_G p_G - \theta_L q) \leq p_G - p_B \leq \bar{\alpha}_G(\beta_G p_G - \theta_L q)$. However, this cannot hold given (RN_G) and $\bar{\alpha}_G < \bar{\alpha}_B$. Therefore, **RN** is infeasible.

• **NN.** When no customer will exercise the refund option, the analysis follows the single-product (No Return) problem. By (A3) the optimal profit is

$$\Pi^{NN} = \max\{\frac{\rho_G\theta_G^2}{2}, \frac{\theta_B^2}{2}\}.$$

We next compare the optimal profit among across all possible types of of products:

$$\Pi^{RR} = \frac{\alpha\theta_H^2}{2}$$
$$\Pi^{NR} = \frac{(\alpha\theta_H + \bar{\rho}\bar{\alpha}\theta_L)^2}{2(1-\rho\bar{\alpha})}$$
$$\Pi^{NN} = \max\{\frac{\rho_G\theta_G^2}{2}, \frac{\theta_B^2}{2}\}.$$

By Lemma A1, if $\frac{\theta_L}{\theta_H} \le \phi(\sqrt{\rho_G}, \alpha_B, \alpha_G)$, $\Pi^{NN} = \frac{\rho_G\theta_G^2}{2}$. Note that $\alpha_G \le \frac{\alpha}{1-\rho\bar{\alpha}} \le 1$, then $\Pi^{NN} = \frac{\rho_G\theta_G^2}{2} \le \Pi^{NR} = \frac{(1-\rho\bar{\alpha})}{2}\mathsf{E}_{\frac{\alpha}{1-\rho\bar{\alpha}}}[\theta]$. Thus,

- **NR** is optimal if $\phi(\sqrt{\frac{\alpha}{1-\rho\bar{\alpha}}}, \frac{\alpha}{1-\rho\bar{\alpha}}, 1) \le \frac{\theta_L}{\theta_H}$,
- **RR** is optimal if $\frac{\theta_L}{\theta_H} \le \phi(\sqrt{\frac{\alpha}{1-\rho\bar{\alpha}}}, \frac{\alpha}{1-\rho\bar{\alpha}}, 1)$.

On the other hand, if $\frac{\theta_L}{\theta_H} \ge \phi(\sqrt{\rho_G}, \alpha_B, \alpha_G)$, $\Pi^{NN} = \frac{\theta_B^2}{2}$. In addition, $\alpha_B \le \frac{\alpha}{1-\rho\bar{\alpha}} \le 1$.

- **NN** is optimal if $\frac{\theta_L}{\theta_H} \ge \max\left\{\phi(\sqrt{\alpha}, \alpha_B, 1), \phi(\sqrt{1-\rho\bar{\alpha}}, \alpha_B, \frac{\alpha}{1-\rho\bar{\alpha}})\right\}$,
- **NR** is optimal if $\phi(\sqrt{\frac{\alpha}{1-\rho\bar{\alpha}}}, \frac{\alpha}{1-\rho\bar{\alpha}}, 1) \le \frac{\theta_L}{\theta_H} \le \phi(\sqrt{1-\rho\bar{\alpha}}, \alpha_B, \frac{\alpha}{1-\rho\bar{\alpha}})$,
- **RR** is optimal if $\frac{\theta_L}{\theta_H} \le \min\left\{\phi(\sqrt{\frac{\alpha}{1-\rho\bar{\alpha}}}, \frac{\alpha}{1-\rho\bar{\alpha}}, 1), \phi(\sqrt{1-\rho\bar{\alpha}}, \alpha_B, \frac{\alpha}{1-\rho\bar{\alpha}})\right\}$.

■

Proof of Proposition 3

For each type of possible products, we first characterize optimal pricing and refund decisions (p_G, p_B, β) under given quality levels (q_G, q_B). The optimal quality design is analyzed thereafter.

• **RR.** When all customers will exercise the refund option, the problem for the firm can be formulated as follows

$$\max_{p_G, p_B, \beta} \Pi^{RR} = \rho_G(p_G - \bar{\alpha}_G\beta p_G - \alpha_G\frac{q_G^2}{2}) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta p_B - \alpha_B\frac{q_B^2}{2})$$
$$\text{s.t.} \quad \begin{array}{lll} u_{GG} & \ge & 0 & (IR_G) \\ u_{BB} & \ge & 0 & (IR_B) \\ u_{GG} & \ge & u_{GB} & (IC_G) \\ u_{BB} & \ge & u_{BG} & (IC_B) \\ \beta p_G & \ge & \theta_L q_G & (RR_G) \\ \beta p_B & \ge & \theta_L q_B & (RR_B) \\ \beta & \le & 1 & , \end{array}$$

where $U_{GG} = -p_G + \bar{\alpha}_G\beta p_G + \alpha_G\theta_H q_G$, $U_{BB} = -p_B + \bar{\alpha}_B\beta p_B + \alpha_B\theta_H q_B$, $U_{GB} = U_{GG} + (1-\bar{\alpha}_G\beta)(p_G - p_B) - \alpha_G\theta_H(q_G - q_B)$, and $U_{BG} = U_{BB} - (1-\bar{\alpha}_B\beta)(p_G - p_B) + \alpha_B\theta_H(q_G - q_B)$. The two IC constraints imply that $\frac{\alpha_B}{1-\bar{\alpha}_B\beta}\theta_H(q_G - q_B) \le p_G - p_B \le \frac{\alpha_G}{1-\bar{\alpha}_G\beta}\theta_H(q_G - q_B)$. As $\frac{\alpha_B}{1-\bar{\alpha}_B\beta} \le \frac{\alpha_G}{1-\bar{\alpha}_G\beta}$, there should be $p_G \ge p_B$ and $q_G \ge q_B$. In addition, high-type customers are always better off than low-type customers under the same product, i.e., $U_{GB} \ge U_{BB}$ and $U_{GG} \ge U_{BG}$. Therefore, $U_{GG} \ge U_{GB} \ge U_{BB} \ge U_{BG}$.

Without affecting the optimal solution, the objective function can be re-written as $\min_{p_G,p_B,\beta} \rho_G U_{GG} + \bar{\rho}_G U_{BB}$. Apparently, the optimum is achieved when (IR_B) and (IC_G) are binding. The problem can then be simplified as

$$\max_{p_G,p_B,\beta} \Pi^{RR} = \rho_G(p_G - \bar{\alpha}_G\beta p_G - \alpha_G\frac{q_G^2}{2}) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta p_B - \alpha_B\frac{q_B^2}{2})$$

$$\text{s.t.} \quad \begin{array}{lcll} p_B - \bar{\alpha}_B\beta p_B - \alpha_B\theta_H q_B & = & 0 & (IR_B) \\ p_G - \bar{\alpha}_G\beta p_G - \alpha_G\theta_H q_G & = & p_B - \bar{\alpha}_G\beta p_B - \alpha_G\theta_H q_B & (IC_G) \\ \beta p_G & \geq & \theta_L q_G & (RR_G) \\ \beta p_B & \geq & \theta_L q_B & (RR_B) \\ \beta & \leq & 1 & \end{array}$$

By (IR_B), $p_B^* = \frac{\alpha_B}{1-\bar{\alpha}_B\beta^*}\theta_H q_B$. Substitute this into (IC_G), we have

$$p_G^* - \bar{\alpha}_G\beta^* p_G^* = \alpha_G\theta_H q_G - \alpha_G\theta_H q_B + \frac{1-\bar{\alpha}_G\beta^*}{1-\bar{\alpha}_B\beta^*}\alpha_B\theta_H q_B.$$

Since $\Pi^* = (\rho\alpha + \bar{\rho}\bar{\alpha})(p_G^* - \bar{\alpha}_G\beta^* p_G^* - \alpha_B\frac{q_G^2}{2}) + (\bar{\rho}\alpha + \rho\bar{\alpha})(\alpha_B\theta_H q_B - \alpha_B\frac{q_B^2}{2})$, we should like to maximize the RHS of the above equation. Due to (A2) and $\beta^* \leq 1$, this can be achieved when $\beta^* = 1$. Subsequently, the optimal solution (with superscript RR) is

$$p_G^* = \theta_H q_G,\ \ p_B^* = \theta_H q_B,\ \ \beta^* = 1 \tag{A11}$$

and $\Pi^* = \rho_G(\alpha_G\theta_H q_G - \alpha_G\frac{q_G^2}{2}) + \bar{\rho}_G(\alpha_B\theta_H q_B - \alpha_B\frac{q_B^2}{2})$. Therefore,

$$\begin{aligned} & q_G^{RR} = \theta_H, \quad q_B^{RR} = \theta_H, \\ & p_G^{RR} = \theta_H^2, \quad p_B^{RR} = \theta_H^2, \\ & \beta^{RR} = 1, \\ & \Pi^{RR} = \rho_G\frac{\alpha_G\theta_H^2}{2} + \bar{\rho}_G\frac{\alpha_B\theta_H^2}{2} \end{aligned}$$

- **NR.** When only the bad-signal customers will exercise the refund, the problem can be formulated as follows

$$\max_{p_G,p_B,\beta} \Pi^{NR} = \rho_G(p_G - q_G^2/2) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta_B p_B - \alpha_B q_B^2/2)$$

$$\text{s.t.} \quad \begin{array}{lcll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & u_{GB} & (IC_G) \\ u_{BB} & \geq & u_{BG} & (IC_B) \\ \beta p_G & \leq & \theta_L q_G & (NR_G) \\ \beta p_B & \geq & \theta_L q_B & (NR_B) \\ \beta & \leq & 1 & , \end{array}$$

where $U_{GG} = -p_G + \theta_G q_G$, $U_{BB} = -p_B + \bar{\alpha}_B\beta p_B + \alpha_B\theta_H q_B$, $U_{GB} = -p_B + \bar{\alpha}_G\beta p_B + \alpha_G\theta_H q_B$, and $U_{BG} = -p_G + \theta_B q_G$. Followed by a similar analysis as in RR, $U_{GG} \geq U_{GB} \geq U_{BB} \geq U_{BG}$. Thus, due to the two NR constraints, we can only confirm that (IR_B) should be binding. The problem can be simplified to

$$\max_{p_G, p_B, \beta} \Pi^{NR} = \rho_G(p_G - q_G^2/2) + \bar{\rho}_G(p_B - \bar{\alpha}_B \beta_B p_B - \alpha_B q_B^2/2)$$

$$\text{s.t.} \quad \begin{array}{lcll} p_B & = & \alpha_B \theta_H q_B + \bar{\alpha}_B \beta p_B & (IR_B) \\ p_G & \leq & \theta_G q_G + p_B - \alpha_G \theta_H q_B - \bar{\alpha}_G \beta p_B & (IC_G) \\ \beta p_G & \leq & \theta_L q_G & (NR_G) \\ \beta p_B & \geq & \theta_L q_B & (NR_B) \\ \beta & \leq & 1 & \end{array}$$

By (IR_B) and (NR_B), we have $\beta \geq \frac{\theta_L}{\theta_B}$. Therefore, $p_G \leq \theta_B q_G$. It can be verified that (IC_G) is also satisfied at $p_G = \theta_B q_G$. Thus, the optimal solution is

$$p_G^* = \theta_B q_G, \; p_B^* = \frac{\alpha_B \theta_H}{1 - \bar{\alpha}_B \theta_L / \theta_B} q_B, \; \beta^* = \theta_L / \theta_B \tag{A12}$$

and the expected profit is $\Pi^* = \rho_G(\theta_B q_G - \frac{q_G^2}{2}) + \bar{\rho}_G \alpha_B (\alpha_B \theta_H q_B - \alpha_B \frac{q_B^2}{2})$. Therefore,

$$\begin{aligned} &q_G^{NR} = \theta_B, \quad q_B^{NR} = \theta_H, \\ &p_G^{NR} = \theta_B^2, \quad p_B^{NR} = \frac{\alpha_B \theta_H^2}{1 - \bar{\alpha}_B \theta_L / \theta_B}, \\ &\beta^{NR} = \theta_L / \theta_B, \\ &\Pi^{NR} = \rho_G \frac{\theta_B^2}{2} + \bar{\rho}_G \frac{\alpha_B \theta_H^2}{2} \end{aligned}$$

- **NN.** When no customer will exercise the refund option, the analysis follows the **(No Return)** problem. By (A7) the optimal profit is

$$\Pi^{NN} = \rho_G \frac{\theta_G^2}{2} + \bar{\rho}_G \frac{\theta_0^2}{2}.$$

- **RN.** When only the good-signal customers will exercise the refund, the problem can be formulated by

$$\max_{p_G, p_B, \beta} \Pi^{RN} = \rho_G(p_G - \bar{\alpha}_G \beta p_G - \alpha_G q_G^2/2) + \bar{\rho}_G(p_B - q_B^2/2)$$

$$\text{s.t.} \quad \begin{array}{lcll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & u_{GB} & (IC_G) \\ u_{BB} & \geq & u_{BG} & (IC_B) \\ \beta p_G & \geq & \theta_L q_G & (RN_G) \\ \beta p_B & \leq & \theta_L q_B & (RN_B) \\ \beta & \leq & 1 & , \end{array}$$

where $U_{GG} = -p_G + \alpha_G \theta_H q_G + \bar{\alpha}_G \beta p_G$, $U_{BB} = -p_B + \theta_B q_B$, $U_{GB} = -p_B + \theta_G q_B$, and $U_{BG} = -p_G + \alpha_B \theta_H q_G + \bar{\alpha}_B \beta p_G$. Followed by a similar argument as in other scenarios, $U_{GG} \geq U_{GB} \geq U_{BB} \geq U_{BG}$, and in this case, only (IC_G) can be confirmed binding due to the two RN constraints. The problem can be simplified to:

$$
\begin{array}{llcll}
\max_{p_G,p_B,\beta} \Pi^{RN} & = \rho_G(p_G - \bar{\alpha}_G\beta p_G - \alpha_G q_G^2/2) + \bar{\rho}_G(p_B - q_B^2/2) & & & \\
 & p_B & \leq & \theta_B q_B & (IR_B) \\
 & (1-\bar{\alpha}_G\beta)p_G & = & \alpha_G\theta_H q_G - \theta_G q_B + p_B & (IC_G) \\
\text{s.t.} & \beta p_G & \geq & \theta_L q_G & (RN_G) \\
 & \beta p_B & \leq & \theta_L q_B & (RN_B) \\
 & \beta & \leq & 1 &
\end{array}
$$

It can be verified that the optimal solution is achieved when (IR_B) and (RN_G) are binding, which gives rise to the following solution:

$$
p_G^* = \theta_G q_G - (\theta_G - \theta_B)q_B, \;\; p_B^* = \theta_B q_B, \;\; \beta^* = \theta_L q_G / p_G^* \tag{A13}
$$

The expected profit is $\Pi^* = \rho_G(\alpha_G\theta_H q_G - \theta_G q_B + \theta_B q_B - \alpha_G \frac{q_G^2}{2}) + \bar{\rho}_G(\theta_B q_B - \frac{q_B^2}{2})$. Thus,

$$
\begin{aligned}
& q_G^{RN} = \theta_H, \quad q_B^{RN} = \theta_0, \\
& p_G^{RN} = (\theta_H - \theta_0)\theta_G + \theta_0\theta_B, \quad p_B^{RN} = \theta_0\theta_B, \\
& \beta^{RN} = \theta_L q_G / p_G^{RN}, \\
& \Pi^{RN} = \rho_G \frac{\alpha_G\theta_H^2}{2} + \bar{\rho}_G \frac{\theta_0^2}{2}
\end{aligned}
$$

where $\theta_0 = \alpha_0\theta_H + (1-\alpha_0)\theta_L$ and $\alpha_0 = \frac{\alpha_B - \rho\alpha}{\bar{\rho}_G} \leq \alpha_B \leq \alpha_G$.

We next compare the optimal profit among across all four types of products:

$$
\begin{aligned}
\Pi^{RR} & = \rho_G \frac{\alpha_G\theta_H^2}{2} + \bar{\rho}_G \frac{\alpha_B\theta_H^2}{2} \\
\Pi^{NR} & = \rho_G \frac{\theta_B^2}{2} + \bar{\rho}_G \frac{\alpha_B\theta_H^2}{2} \\
\Pi^{NN} & = \rho_G \frac{\theta_G^2}{2} + \bar{\rho}_G \frac{\theta_0^2}{2} = \frac{\theta_B^2}{2} + \frac{\rho_G}{2\bar{\rho}_G}(\theta_G - \theta_B)^2 \\
\Pi^{RN} & = \rho_G \frac{\alpha_G\theta_H^2}{2} + + \bar{\rho}_G \frac{\theta_0^2}{2}.
\end{aligned}
$$

We find that

- if $\sqrt{\alpha_G} \leq \frac{\theta_B}{\theta_H}$, $\Pi^{RR} \leq \Pi^{NR}$ and $\Pi^{RN} \leq \Pi^{NN}$. Moreover, $\Pi^{NR} \leq \frac{\theta_B^2}{2} \leq \Pi^{NN}$. Therefore, it is better to retreat to (No Return) products.
- if $\frac{\theta_B}{\theta_H} < \sqrt{\alpha_G} \leq \frac{\theta_G}{\theta_H}$, there is $\Pi^{RR} > \Pi^{NR}$ and $\Pi^{RN} \leq \Pi^{NN}$. In this scenario, however, there is $\Pi^{RR} \leq \frac{\theta_G^2}{2} \leq \Pi^{NN}$. Again, better to retreat to (No Return) products.

- if $\frac{\theta_G}{\theta_H} < \sqrt{\alpha_G}$, $\Pi^{RR} > \Pi^{NR}$ and $\Pi^{RN} > \Pi^{NN}$. We only need to consider **RR** and **RN**. Specifically, if $\frac{\theta_0}{\theta_H} < \sqrt{\alpha_B}$, **RR** is optimal; otherwise **RN** is optimal.

By Lemma A1, **NN** is optimal if $\frac{\theta_L}{\theta_H} \geq \phi(\sqrt{\alpha_G}, \alpha_G, 1)$.

If $\frac{\theta_L}{\theta_H} \leq \phi(\sqrt{\alpha_G}, \alpha_G, 1)$, **RR** is optimal if $\frac{\theta_L}{\theta_H} \leq \phi(\sqrt{\alpha_B}, \alpha_0, 1)$ or $\frac{\theta_L}{\theta_H} \geq \phi(\sqrt{\alpha_G}, \alpha_0, 1)$. Otherwise, **RN** is optimal.

■

Proof of Theorem 1

- **RR.** When both "Good" and "Bad" signalled customers exercise the refund option, the problem for the firm can be formulated as follows

$$\max_{p_G,p_B,\beta_G,\beta_B} \Pi^{RR} = \rho_G(p_G - \bar{\alpha}_G\beta_G p_G - \alpha_G q_G^2/2) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta_B p_B - \alpha_B q_B^2/2)$$
$$\text{s.t.} \quad \begin{array}{lll} u_{GG} & \geq 0 & (IR_G) \\ u_{BB} & \geq 0 & (IR_B) \\ u_{GG} & \geq u_{GB} & (IC_G) \\ u_{BB} & \geq u_{BG} & (IC_B) \\ \beta_G p_G & \geq \theta_L q_G & (RR_G) \\ \beta_B p_B & \geq \theta_L q_B & (RR_B) \\ \beta_G & \leq 1 & \\ \beta_B & \leq 1 & \end{array}$$

where $U_{GG} = -p_G + \bar{\alpha}_G\beta_G p_G + \alpha_G\theta_H q_G$, $U_{BB} = -p_B + \bar{\alpha}_B\beta_B p_B + \alpha_B\theta_H q_B$, $U_{GB} = U_{GG} + (1 - \bar{\alpha}_G\beta_G)p_G - (1 - \bar{\alpha}_G\beta_B)p_B - \alpha_G\theta_H(q_G - q_B)$, and $U_{BG} = U_{BB} - (1 - \bar{\alpha}_B\beta_G)p_G + (1 - \bar{\alpha}_B\beta_B)p_B + \alpha_B\theta_H(q_G - q_B)$. The IC constraints essentially require that

$$\frac{1 - \bar{\alpha}_G\beta_G}{\alpha_G}p_G - \frac{1 - \bar{\alpha}_G\beta_B}{\alpha_G}p_B \leq \theta_H(q_G - q_B) \leq \frac{1 - \bar{\alpha}_B\beta_G}{\alpha_B}p_G - \frac{1 - \bar{\alpha}_B\beta_B}{\alpha_B}p_B$$

By (A1), the above implies that $p_G \leq p_B$. In addition, as high-type customers are always better off than low-type customers under the same product, i.e., $U_{GB} \geq U_{BB}$ and $U_{GG} \geq U_{BG}$, there should be $U_{GG} \geq U_{GB} \geq U_{BB} \geq U_{BG}$.

Without affecting the optimal solution, the objective function can be re-written as $\min_{p_G,p_B,\beta} \rho_G U_{GG} + \bar{\rho}_G U_{BB}$. Hence the optimal solution should be binding at (IR_B) and (IC_G). The problem can be simplified to:

$$\max_{p_G,p_B,\beta_G,\beta_B} \Pi^{RR} = \rho_G(p_G - \bar{\alpha}_G\beta_G p_G - \alpha_G q_G^2/2) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta_B p_B - \alpha_B q_B^2/2)$$
$$\text{s.t.} \quad \begin{array}{llll} p_B - \bar{\alpha}_B\beta_B p_B & = & \alpha_B\theta_H q_B & (IR_B) \\ p_G - \bar{\alpha}_G\beta_G p_G & = & \alpha_G\theta_H q_G - \alpha_G\theta_H q_B + p_B - \bar{\alpha}_G\beta_B p_B & (IC_G) \\ \beta_G p_G & \geq & \theta_L q_G & (RR_G) \\ \beta_B p_B & \geq & \theta_L q_B & (RR_B) \\ \beta_G & \leq & 1 & \\ \beta_B & \leq & 1 & \end{array}$$

In maximizing the objective function, we need to maximize the RHS of (IC_G), which equals to $\alpha_G\theta_H q_G - \alpha_G\theta_H q_B + \alpha_B\theta_H q_B + (\alpha_G - \alpha_B)\beta_B p_B$. By (IR_B), there should be $\beta_B^* = 1$ and $p_B^* = \theta_H q_B$. Overall,

$$p_G^* \in [\alpha_G\theta_H q_G, \theta_H q_G],\ \ \beta_G^* = \frac{1}{\bar{\alpha}_G} - \frac{\alpha_G}{\bar{\alpha}_G}\frac{\theta_H q_G}{p_G},\ \ p_B^* = \theta_H q_B,\ \ \beta_B^* = 1 \tag{A14}$$

In particular, these include the following products:

$$p_G^* = \theta_G q_G,\ \ \beta_G^* = \theta_L/\theta_G,\ \ p_B^* = \theta_H q,\ \ \beta_B^* = 1;$$
$$p_G^* = \theta_H q_G,\ \ \beta_G^* = 1,\ \ p_B^* = \theta_H q,\ \ \beta_B^* = 1.$$

The expected profit is: $\Pi^* = \rho_G(\alpha_G\theta_H q_G - \alpha_G q_G^2/2) + \bar{\rho}_G(\alpha_B\theta_H q_B - \alpha_B q_B^2/2)$. Therefore, the optimal quality is

$$\begin{aligned}
&q_G^{RR} = \theta_H, \quad q_B^{RR} = \theta_H, \\
&p_G^{RR} \in [\alpha_G\theta_H q_G, \theta_H q_G], \quad p_B^{RR} = \theta_H^2, \\
&\beta_G^{RR} = \frac{1}{\bar{\alpha}_G} - \frac{\alpha_G}{\bar{\alpha}_G}\frac{\theta_H q_G^{RR}}{p_G^{RR}}, \beta_B^{RR} = 1, \\
&\Pi^{RR} = \rho_G\frac{\alpha_G\theta_H^2}{2} + \bar{\rho}_G\frac{\alpha_B\theta_H^2}{2}
\end{aligned}$$

- **NR.** When only the "Bad" signalled customer will exercise the refund, i.e., $\beta_G = 0$, the problem can be formulated as follows

$$\begin{array}{rrcll}
\max\limits_{p_G, p_B, \beta_B} \Pi^{NR} & = \rho_G(p_G - q_G^2/2) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta_B p_B - \alpha_B q_B^2/2) & & & \\
& u_{GG} & \geq & 0 & (IR_G) \\
& u_{BB} & \geq & 0 & (IR_B) \\
\text{s.t.} & u_{GG} & \geq & u_{GB} & (IC_G) \\
& u_{BB} & \geq & u_{BG} & (IC_B) \\
& \beta_B p_B & \geq & \theta_L q_B & (NR_B) \\
& \beta_B & \leq & 1 &
\end{array}$$

where $U_{GG} = -p_G + \theta_G q_G$, $U_{BB} = -p_B + \bar{\alpha}_B\beta_B p_B + \alpha_B\theta_H q_B$, $U_{GB} = -p_B + \bar{\alpha}_G\beta_B p_B + \alpha_G\theta_H q_B$, and $U_{BG} = -p_G + \theta_B q_G$. Followed by a similar analysis as in RR, $U_{GG} \geq U_{GB} \geq U_{BB} \geq U_{BG}$. Thus, both (IR_B) and (IC_G) will be binding.The problem can be simplified to

$$\begin{array}{rrcll}
\max\limits_{p_G, p_B, \beta_B} \Pi^{NR} & = \rho_G(p_G - q_G^2/2) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta_B p_B - \alpha_B q_B^2/2) & & & \\
& p_B - \bar{\alpha}_B\beta_B p_B & = & \alpha_B\theta_H q_B & (IR_B) \\
\text{s.t.} & p_G & = & \theta_G q_G + p_B - \alpha_G\theta_H q_B - \bar{\alpha}_G\beta_B p_B & (IC_G) \\
& \beta_B p_B & \geq & \theta_L q & (NR_L) \\
& \beta_B & \leq & 1 &
\end{array}$$

The optimum can be obtained as follows:

$$p_G^* = \theta_G q_G,\ \ \beta_G^* = 0,\ \ p_B^* = \theta_H q_B,\ \ \beta_B^* = 1. \tag{A16}$$

and the expected profit is $\Pi^* = \rho_G(\theta_G q_G - q_G^2/2) + \bar{\rho}_G(\alpha_B \theta_H q_B - \alpha_B q_B^2/2)$. Therefore, the optimal quality is

$$\begin{aligned}
&q_G^{NR} = \theta_G, \quad q_B^{NR} = \theta_H,\\
&p_G^{NR} = \theta_G^2, \quad p_B^{NR} = \theta_H^2,\\
&\beta_G^{NR} = 0, \ \ \beta_B^{NR} = 1,\\
&\Pi^{NR} = \rho_G \frac{\theta_G^2}{2} + \bar{\rho}_G \frac{\alpha_B \theta_H^2}{2}
\end{aligned}$$

- **RN.** When the refund only goes to "Good" signalled customers, i.e., $\beta_B = 0$, the following problem needs to be solved:

$$\begin{array}{rrcll}
\max\limits_{p_G, p_B, \beta_G} \Pi^{RN} & = & \multicolumn{3}{l}{\rho_G(p_G - \bar{\alpha}_G \beta_G p_G - \alpha_G q_G^2/2) + \bar{\rho}_G(p_B - q_B^2/2)} \\
 & u_{GG} & \geq & 0 & (IR_G) \\
 & u_{BB} & \geq & 0 & (IR_B) \\
\text{s.t.} & u_{GG} & \geq & u_{GB} & (IC_G) \\
 & u_{BB} & \geq & u_{BG} & (IC_B) \\
 & \beta_G p_G & \geq & \theta_L q_G & (RN_G) \\
 & \beta_G & \leq & 1 &
\end{array}$$

where $U_{GG} = -p_G + \alpha_G \theta_H q_G + \bar{\alpha}_G \beta_G p_G$, $U_{BB} = -p_B + \theta_B q_B$, $U_{GB} = -p_B + \theta_G q_B$, and $U_{BG} = -p_G + \alpha_B \theta_H q_G + \bar{\alpha}_B \beta_G p_G$. Followed by a similar argument as other scenarios, $U_{GG} \geq U_{GB} \geq U_{BB} \geq U_{BG}$, both (IR_B) and (IC_G) are binding:

$$\begin{array}{rrcll}
\max\limits_{p_G, p_B, \beta_G} \Pi^{RN} & = & \multicolumn{3}{l}{\rho_G(p_G - \bar{\alpha}_G \beta_G p_G - \alpha_G q_G^2/2 + \bar{\rho}_G(p_B - q_B^2/2)} \\
 & p_B & = & \theta_B q_B & (IR_B) \\
\text{s.t.} & p_G & = & \alpha_G \theta_H q_G + \bar{\alpha}_G \beta_G p_G + p_B - \theta_G q_B & (IC_G) \\
 & \beta_G p_G & \geq & \theta_L q_G & (RN_H) \\
 & \beta_G & \leq & 1 &
\end{array}$$

It can be verified that the optimal menu would be of the following form:

$$\begin{aligned}
&p_G^* \in [\theta_G q_G - (\theta_G - \theta_B) q_B, \theta_B q_B + \theta_H (q_G - q_B)],\ \ \beta_G^* = \frac{1}{\bar{\alpha}_G} - \frac{\alpha_G \theta_H q_G - (\theta_G - \theta_B) q_B}{\bar{\alpha}_G p_G^*}\\
&p_B^* = \theta_B q_B,\ \ \beta_B^* = 0.
\end{aligned}$$

The optimal expected profit is then $\Pi^* = \rho_G[\alpha_G \theta_H q_G - (\theta_G - \theta_B) q_B - \alpha_G q_G^2/2] + \bar{\rho}_G(\theta_B q_B - q_B^2/2)$. Thus,

$$q_G^{RN} = \theta_H, \quad q_B^{RN} = \theta_0,$$

$$p_G^{RN} = \in [\theta_G\theta_H - (\theta_G - \theta_B)\theta_0, \theta_0\theta_B + \theta_H(\theta_H - \theta_0)], \quad p_B^{RN} = \theta_0\theta_B,$$
$$\beta_G^{RN} = \frac{1}{\bar{\alpha}_G} - \frac{\alpha_G\theta_H q_G^{RN} - (\theta_G - \theta_B)\theta_0}{\bar{\alpha}_G p_G^{RN}}, \quad \beta_B^{RN} = 0,$$
$$\Pi^{RN} = \rho_G \frac{\alpha_G\theta_H^2}{2} + \bar{\rho}_G \frac{\theta_0^2}{2}$$

where $\theta_0 = \alpha_0\theta_H + (1-\alpha_0)\theta_L$ and $\alpha_0 = \frac{\alpha_B - \rho\alpha}{\bar{\rho}_G} \le \alpha_B \le \alpha_G$.

• **NN.** When no customer will exercise the refund, analysis can be retrieved to the **(No Return)** problem and the optimal profit is by (A7) is

$$\Pi^{NN} = \rho_G \frac{\theta_G^2}{2} + \bar{\rho}_G \frac{\theta_0^2}{2}.$$

Comparing the optimal profits under four kinds of products:

$$\Pi^{RR} = \rho_G \frac{\alpha_G\theta_H^2}{2} + \bar{\rho}_G \frac{\alpha_B\theta_H^2}{2}$$
$$\Pi^{NR} = \rho_G \frac{\theta_G^2}{2} + \bar{\rho}_G \frac{\alpha_B\theta_H^2}{2}$$
$$\Pi^{RN} = \rho_G \frac{\alpha_G\theta_H^2}{2} + \bar{\rho}_G \frac{\theta_0^2}{2}$$
$$\Pi^{NN} = \rho_G \frac{\theta_G^2}{2} + \bar{\rho}_G \frac{\theta_0^2}{2}$$

By Lemma A1, **NN** is optimal product if $\frac{\theta_L}{\theta_H} \ge \max\{\phi(\sqrt{\alpha_G}, \alpha_G, 1), \phi(\sqrt{\alpha_B}, \alpha_0, 1)\}$.
NR is optimal if $\phi(\sqrt{\alpha_G}, \alpha_G, 1) \le \frac{\theta_L}{\theta_H} \le \phi(\sqrt{\alpha_B}, \alpha_0, 1)$.
RN is optimal if $\phi(\sqrt{\alpha_B}, \alpha_0, 1) \le \frac{\theta_L}{\theta_H} \le \phi(\sqrt{\alpha_G}, \alpha_G, 1)$.
RR is optimal if $\frac{\theta_L}{\theta_H} \le \min\{\phi(\sqrt{\alpha_G}, \alpha_G, 1), \phi(\sqrt{\alpha_B}, \alpha_0, 1)\}$.

■

Proof of Corollary 1

(i) The statement follows immediately after Theorem 1 (i) where $q_G^* = q_B^*$.
(ii) The statement follows immediately after Theorem 1 (i), where $\beta_G = \beta_B = 1$ constitutes an optimum, and (iii) where $\beta_G^* = \beta_B^* = 0$.
(iii) By Theorem 1 and (ii), standard refund universally applies if the market outcome is **RN** when valuation heterogeneity level is moderate, or equivalently $\phi(\sqrt{\alpha_B}, \alpha_0, 1) \le \phi(\sqrt{\alpha_G}, \alpha_G, 1)$, as suggested by end of proof of Theorem 1. By definition, $\phi(\sqrt{\alpha_G}, \alpha_G, 1) - \phi(\sqrt{\alpha_B}, \alpha_0, 1) = \frac{\sqrt{\alpha_G} - \alpha_G}{1-\alpha_G} - \frac{\sqrt{\alpha_B} - \alpha_0}{1-\alpha_0} = \frac{1-\sqrt{\alpha_B}}{1-\alpha_0} - \frac{1-\sqrt{\alpha_G}}{1-\alpha_G}$. Therefore, $\phi(\sqrt{\alpha_G}, \alpha_G, 1) \ge \phi(\sqrt{\alpha_B}, \alpha_0, 1)$ is equivalent to $\frac{1-\sqrt{\alpha_B}}{1-\alpha_0} \ge \frac{1-\sqrt{\alpha_G}}{1-\alpha_G} = \frac{1}{1+\sqrt{\alpha_G}}$. After some algebra, the condition for standard refund to be optimal is

$$S(\rho, \alpha) = -\sqrt{\alpha_B\alpha_G} - \sqrt{\alpha_B} + \sqrt{\alpha_G} + \alpha_0 \ge 0.$$

We can show that for any given α, there exists a $\hat{\rho} \in [0.5, 1]$ such that $S \leq 0$ when $\rho \leq \hat{\rho}$ and $S \geq 0$ when $\rho \geq \hat{\rho}$.

- First consider $\alpha \in [0, 0.5]$. Under this condition, there are $\bar{\alpha} \geq \alpha$ and $\bar{\rho}_G \geq \rho_G$. At $\rho = 0.5$, we have $\alpha_B = \alpha_G = \alpha_0 = \alpha$ and $S(0.5, \alpha) = 0$. We next prove that S is convex in ρ, i.e.,

$$\frac{\partial^2 S}{\partial \rho^2} = -\frac{\partial^2 \sqrt{\alpha_B \alpha_G}}{\partial \rho^2} - \frac{\partial^2 \sqrt{\alpha_B}}{\partial \rho^2} + \frac{\partial^2 \sqrt{\alpha_G}}{\partial \rho^2} + \frac{\partial^2 \alpha_0}{\partial \rho^2} \geq 0 \tag{A17}$$

by showing that

(a) $\frac{\partial^2 \alpha_0}{\partial \rho^2} \geq 0$,

(b) $\frac{\partial^2 \sqrt{\alpha_G}}{\partial \rho^2} - \frac{\partial^2 \sqrt{\alpha_B}}{\partial \rho^2} \geq 0$, and

(c) $\frac{\partial^2 \sqrt{\alpha_G \alpha_B}}{\partial \rho^2} \leq 0$.

As a preliminary, it can be verified that

$$\frac{\partial \alpha_G}{\partial \rho} = \frac{\alpha \bar{\alpha}}{\rho_G^2}, \quad \frac{\partial \alpha_B}{\partial \rho} = -\frac{\alpha \bar{\alpha}}{\bar{\rho}_G^2}, \quad \frac{\partial \alpha_0}{\partial \rho} = \frac{\alpha \bar{\alpha}}{\bar{\rho}_G^2} - \frac{2\alpha \bar{\alpha}}{\bar{\rho}_G^3}$$
$$\frac{\partial^2 \alpha_G}{\partial \rho^2} = \frac{2\alpha \bar{\alpha}(\bar{\alpha} - \alpha)}{\rho_G^3}, \quad \frac{\partial^2 \alpha_B}{\partial \rho^2} = \frac{2\alpha \bar{\alpha}(\bar{\alpha} - \alpha)}{\bar{\rho}_G^3}, \quad \frac{\partial^2 \alpha_0}{\partial \rho^2} = -\frac{2\alpha \bar{\alpha}(\bar{\alpha} - \alpha)}{\bar{\rho}_G^3} + \frac{6\alpha \bar{\alpha}(\bar{\alpha} - \alpha)}{\bar{\rho}_G^4}$$

(a) apparently holds by the derivatives above.

To prove (b), it can be obtained that $\frac{\partial^2 \sqrt{\alpha_G}}{\partial \rho^2} - \frac{\partial^2 \sqrt{\alpha_B}}{\partial \rho^2} = \frac{1}{\rho^{3/2} \alpha^{1/2} \rho_G^{5/2}}[2(\bar{\alpha} - \alpha)\rho - \bar{\alpha}] - \frac{1}{\bar{\rho}^{3/2} \alpha^{1/2} \bar{\rho}_G^{5/2}}[2(\bar{\alpha} - \alpha)\bar{\rho} - \bar{\alpha}]$. Note that $2(\bar{\alpha} - \alpha)\bar{\rho} - \bar{\alpha} \leq 0$. When $2(\bar{\alpha} - \alpha)\rho - \bar{\alpha} \geq 0$, apparently there is $\frac{\partial^2 \sqrt{\alpha_G}}{\partial \rho^2} - \frac{\partial^2 \sqrt{\alpha_B}}{\partial \rho^2} \geq 0$. Otherwise, suppose $2(\bar{\alpha} - \alpha)\rho - \bar{\alpha} \leq 0$. Consider a function $T(y) = \frac{1}{y^{3/2}(y\alpha + (1-y)\bar{\alpha})^{5/2}}[2(\bar{\alpha} - \alpha)y - \bar{\alpha}]$. After some algebra, we have $\frac{\partial T(y)}{\partial y} = \frac{3\bar{\alpha} - 6(\bar{\alpha} - \alpha)y}{y^{5/2}(y\alpha + (1-y)\bar{\alpha})^{5/2}} \geq 0$ for $y \in [0, \frac{\bar{\alpha}}{2(\bar{\alpha} - \alpha)}]$. Therefore, $T(y)$ increases on the interval satisfying $2(\bar{\alpha} - \alpha)y - \bar{\alpha} \leq 0$, which includes $[\bar{\rho}, \rho]$. Hence there is also $\frac{\partial^2 \sqrt{\alpha_G}}{\partial \rho^2} - \frac{\partial^2 \sqrt{\alpha_B}}{\partial \rho^2} = \alpha^{-1/2}[T(\rho) - T(\bar{\rho})] \geq 0$.

For (c), after some algebra, the second derivative of $\sqrt{\alpha_G \alpha_B}$ can be reduced to $\frac{\partial^2 \sqrt{\alpha_G \alpha_B}}{\partial \rho^2} = \frac{\alpha^2 \bar{\alpha}^2}{4(\alpha_B \alpha_G)^{3/2}}[-(\frac{\alpha_G}{\bar{\rho}_G^2} + \frac{\alpha_B}{\rho_G^2})^2 + 4\alpha_B \alpha_G \frac{\bar{\alpha} - \alpha}{\alpha \bar{\alpha}}(\frac{\alpha_G}{\bar{\rho}_G^3} + \frac{\alpha_B}{\rho_G^3})]$. Note that $\frac{\bar{\alpha} - \alpha}{\alpha \bar{\alpha}}(\frac{\alpha_G}{\bar{\rho}_G^3} + \frac{\alpha_B}{\rho_G^3})\rho_G^2 \bar{\rho}_G^2 = \frac{\bar{\alpha} - \alpha}{\alpha \bar{\alpha}}(\frac{\alpha_G \rho_G^2}{\bar{\rho}_G} + \frac{\alpha_B \bar{\rho}_G^2}{\rho_G}) = \frac{\bar{\alpha} - \alpha}{\bar{\alpha}}(\frac{\rho \rho_G}{\bar{\rho}_G} + \frac{\bar{\rho} \bar{\rho}_G}{\rho_G})$. Since $\rho_G \leq \bar{\rho}_G$ and $\frac{\rho}{\bar{\rho}_G} \geq \frac{\bar{\rho}}{\rho_G}$, there is $\frac{\rho \rho_G}{\bar{\rho}_G} + \frac{\bar{\rho} \bar{\rho}_G}{\rho_G} \leq \frac{\rho \bar{\rho}_G}{\bar{\rho}_G} + \frac{\bar{\rho} \rho_G}{\rho_G} = 1$. Thus $\frac{\bar{\alpha} - \alpha}{\alpha \bar{\alpha}}(\frac{\alpha_G}{\bar{\rho}_G^3} + \frac{\alpha_B}{\rho_G^3})\rho_G^2 \bar{\rho}_G^2 = \frac{\bar{\alpha} - \alpha}{\bar{\alpha}}(\frac{\rho \rho_G}{\bar{\rho}_G} + \frac{\bar{\rho} \bar{\rho}_G}{\rho_G}) \leq \frac{\bar{\alpha} - \alpha}{\bar{\alpha}} \leq 1$. Therefore, $\frac{\partial^2 \sqrt{\alpha_G \alpha_B}}{\partial \rho^2} \leq \frac{\alpha^2 \bar{\alpha}^2}{4(\alpha_B \alpha_G)^{3/2}}[-(\frac{\alpha_G}{\bar{\rho}_G^2} + \frac{\alpha_B}{\rho_G^2})^2 + 4\frac{\alpha_B \alpha_G}{\rho_G^2 \bar{\rho}_G^2}] \leq 0$.

- When $\alpha \in [0.5, 1]$, there are $\bar{\alpha} \leq \alpha$ and $\bar{\rho}_G \leq \rho_G$. We can prove that a standard refund is never optimal in this region by showing that S is negative for any $\rho \in [0.5, 1]$ thus $\hat{\rho} = 1$. After some algebra, $S(\rho, \alpha) \geq 0$ becomes equivalent to

$$\sqrt{\alpha_G} - \sqrt{\alpha_B} \geq \frac{\alpha_B - \alpha_0}{1 - \sqrt{\alpha_B}}. \tag{A18}$$

By noting that $\alpha_B - \alpha_0 = \frac{\alpha - \alpha_B}{\bar{\rho}_G}$ and $\alpha_G - \alpha_B = \frac{\alpha - \alpha_B}{\rho_G}$, the above condition becomes $\bar{\rho}_G \geq \sqrt{\alpha_B} + \rho_G\sqrt{\alpha_G}$. However, since $\bar{\rho}_G = \bar{\rho}\alpha + \rho\bar{\alpha} \leq \frac{\bar{\rho}\alpha}{\bar{\rho}_G} + \rho\alpha = \alpha_B + \rho_G\alpha_G \leq \sqrt{\alpha_B} + \rho_G\sqrt{\alpha_G}$, (A18) does not hold. Therefore $S \leq 0$ whenever $\alpha \geq 0.5$ and the firm should turn to customized refund in maximizing her profit.

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Proof of Proposition 5

LEMMA A3.

(i) $\frac{\partial \Pi^{RR}}{\partial \rho} = 0$ *and* $\frac{\partial^2 \Pi^{NR}}{\partial \rho^2} \geq 0$*;*

(ii) $\frac{\partial^2 \Pi^{RN}}{\partial \rho^2} \geq 0$ *when* $\rho \to 1$ *and* $\alpha < \frac{\theta_L}{3\theta_H}$ *or* $\alpha > \theta_L/\theta_H$*;*

(iii) $\frac{\partial \Pi^{NN}}{\partial \rho} > 0$ *when* $\rho \to 1$ *and* $\alpha > \frac{\theta_L}{\theta_H}$*;* $\frac{\partial \Pi^{NN}}{\partial \rho} < 0$ *when* $\rho \to 0.5$*.*

Proof. (i) The first order derivative with Π^{RR} is apparent. For Π^{NR}, it can be proved that $\frac{\partial^2 \Pi^{NR}}{\partial \rho^2} \sim \frac{\alpha^2\bar{\alpha}^2(\theta_H - \theta_L)^2}{\rho_G^3} \geq 0$.

(ii) Note that $\frac{\partial \rho_G}{\partial \rho} = 2\alpha - 1$, $\frac{\partial \alpha_G}{\partial \rho} = \frac{\alpha\bar{\alpha}}{\rho_G^2}$, $\frac{\partial \alpha_0}{\partial \rho} = \frac{\alpha\bar{\alpha}}{\bar{\rho}_G^2} - \frac{2\alpha\bar{\alpha}}{\bar{\rho}_G^3} = -\frac{\alpha\bar{\alpha}(1+\rho_G)}{\bar{\rho}_G^3}$. Thus $\frac{\partial \Pi^{RN}}{\partial \rho} = [(2\alpha - 1)\alpha_G + \frac{\alpha\bar{\alpha}}{\rho_G}]\theta_H^2 + (1 - 2\alpha)\theta_0^2 - 2\frac{\alpha\bar{\alpha}(1+\rho_G)}{\bar{\rho}_G}\theta_0(\theta_H - \theta_L)$. When $\rho \to 1$, there are $\rho_G \to \alpha$, $\alpha_G \to 1$, and $\alpha_0 \to -\alpha/\bar{\alpha}$. After some algebra, it can be verified that $\lim_{\rho\to 1} \frac{\partial \Pi^{RN}}{\partial \rho} \sim \alpha\theta_H^2 + \frac{(\theta_L - \alpha\theta_H)(\theta_L - 3\alpha\theta_H)}{\bar{\alpha}^2} > 0$ when $\alpha < \frac{\theta_L}{3\theta_H}$ or $\alpha > \theta_L/\theta_H$.

(iii) Note that $\frac{\partial \alpha_G}{\partial \rho} = \frac{\alpha\bar{\alpha}}{\rho_G^2}$, $\frac{\partial \alpha_B}{\partial \rho} = -\frac{\alpha\bar{\alpha}}{\bar{\rho}_G^2}$, $\frac{\partial \alpha_0}{\partial \rho} = \frac{\alpha\bar{\alpha}}{\bar{\rho}_G^2} - \frac{2\alpha\bar{\alpha}}{\bar{\rho}_G^3} = -\frac{\alpha\bar{\alpha}(1+\rho_G)}{\bar{\rho}_G^3}, \alpha_G - \alpha_0 = \frac{\alpha\bar{\alpha}(\rho - \bar{\rho})}{\rho_G\bar{\rho}_G^2}$. Thus

$$\begin{aligned}\frac{\partial \Pi^{NN}}{\partial \rho} &\sim \frac{\alpha - \bar{\alpha}}{2}(\theta_G^2 - \theta_0^2) + (\theta_H - \theta_L)\alpha\bar{\alpha}\left(\frac{\theta_G}{\rho_G} - \frac{\theta_0(1+\rho_G)}{\bar{\rho}_G^2}\right) \\ &= (\theta_H - \theta_L)\left[\theta_G(\frac{(\alpha - \bar{\alpha})(\alpha_G - \alpha_0)}{2} + \frac{\alpha\bar{\alpha}}{\rho_G}) + \theta_0(\frac{(\alpha - \bar{\alpha})(\alpha_G - \alpha_0)}{2} - \frac{\alpha\bar{\alpha}(1+\rho_G)}{\bar{\rho}_G^2})\right]\end{aligned}$$

As $\rho \to 1$, there are $\rho_G \to \alpha$, $\bar{\rho}_G \to \bar{\alpha}$, $\alpha_G \to 1$, $\alpha_B \to 0$ and $\alpha_0 \to -\alpha/\bar{\alpha}$. It can be verified that

$$\begin{aligned}\lim_{\rho\to 1} \frac{\partial \Pi^{NN}}{\partial \rho} &= (\theta_H - \theta_L)\left[\theta_H(\frac{\alpha - \bar{\alpha}}{2\bar{\alpha}} + \bar{\alpha}) + (-\frac{\alpha}{\bar{\alpha}}\theta_H + \frac{1}{\bar{\alpha}}\theta_L)(\frac{\alpha - \bar{\alpha}}{2\bar{\alpha}} - \frac{\alpha(1+\alpha)}{\bar{\alpha}})\right] \\ &= (\theta_H - \theta_L)\left[\theta_H\frac{1 - 2\alpha + 2\alpha^2}{2\bar{\alpha}} + \frac{(\alpha\theta_H - \theta_L)(1 + 2\alpha^2)}{2\bar{\alpha}^2}\right]\end{aligned}$$

which is greater than 0 as long as $\alpha > \theta_L/\theta_H$. Thus for large α, Π^{NN} increases in ρ when it is close to 1.

As $\rho \to 0.5$, there are $\rho_G \to 0.5$, $\bar{\rho}_G \to 0.5$, $\alpha_G \to \alpha$, $\alpha_B \to \alpha$ and $\alpha_0 \to \alpha$.

$$\lim_{\rho\to 0.5} \frac{\partial \Pi^{NN}}{\partial \rho} = (\theta_H - \theta_L)\left[2\theta_G\alpha\bar{\alpha} - 3\theta_0\alpha\bar{\alpha}\right] = -(\theta_H - \theta_L)(\alpha\theta_H + \bar{\alpha}\theta_L)\alpha\bar{\alpha} < 0$$

Therefore, Π^{NN} decreases in ρ when it is close to 0.5. □

Lemma A3 (i) shows that firm's profit is insensitive to signal quality when heterogeneity level is high such that **RR** is the optimal menu. Lemma A3 (ii) and the first part of (iii) together show that signal improvement can possibly rewarding when **RN** and **NN** is the optimal menu with high signal quality; and the second part of (iii) confirms that the firm may have incentive to reduce signal quality when heterogeneity level is low and signal quality is below certain level towards 0.5.

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